

Assignment

a Show that the limit of the function given in equation (1.2) as x approaches 0 is $\frac{a}{b}$

$$f(x) = \frac{a \sin bx}{bx}$$

Soln

$$f(x) = \frac{a \sin x}{bx}$$

$$= \frac{a \sin(x)}{bx} = \frac{0}{0} \text{ (undefined)}$$

Using L'Hopital's rule

$$f(x) = \frac{a \cos x}{b}$$

$$= \frac{a \cos 0}{b}$$

$$\lim_{x \rightarrow 0} f(x) = \frac{a \cos(0)}{b} = \frac{a}{b}$$

b The model of a system has been developed to be as given in eqn. (1.22)
 $f(x) = 5x - 2.7$

Given that $\delta = 0.1$ and $\Delta \delta = 0.01$, demonstrate in a tabular form, that the limit of the model as $x \rightarrow b$ is equal to a

Soln

$$\delta = 0.1, \Delta \delta = 0.01, \epsilon = 0.05$$

$$b - 0.1 = 5.9 \quad \left\{ \begin{array}{l} \text{left hand side} \\ \text{right hand side} \end{array} \right\}$$

$$b + 0.1 = 6.1 \quad \left\{ \begin{array}{l} \text{right hand side} \\ \text{left hand side} \end{array} \right\}$$

$$a - 0.5 = 8.5 \quad \left\{ \begin{array}{l} \text{left hand side} \\ \text{right hand side} \end{array} \right\}$$

$$a + 0.5 = 9.5 \quad \left\{ \begin{array}{l} \text{right hand side} \\ \text{left hand side} \end{array} \right\}$$

x	$f(x)$	x	$f(x)$
5.91	8.5	6.1	9.5
5.92	8.55	6.09	9.45
5.92	8.60	6.08	9.40
5.93	8.65	6.09	9.35
5.94	8.70	6.06	9.30
5.95	8.75	6.05	9.25
5.96	8.80	6.04	9.20
5.97	8.85	6.03	9.15
5.98	8.90	6.02	9.10
5.99	8.95	6.01	9.05
6.00	9.00	6.00	9.00

c) Show whether the ~~function~~ function given in the equation below is continuous on the interval $[-5, 5]$

$$f(x) = (25 - x^2)^{1/2}$$

Soln

$$x \rightarrow -5$$

$$f(x) = (25 - (-5)^2)^{1/2}$$

$$= 0$$

$$x \rightarrow -4$$

$$f(x) = (25 - (-4)^2)^{1/2}$$

$$= 3$$

$$x \rightarrow -3$$

$$f(x) = (25 - (-3)^2)^{1/2}$$

$$= 4$$

$$x \rightarrow -2$$

$$f(x) = (25 - (-2)^2)^{1/2}$$

$$= 4.58$$

$$x \rightarrow -1$$

$$f(x) = (25 - (-1)^2)^{1/2}$$

$$= 4.89$$

$$x \rightarrow 0$$

$$f(x) = (25 - (-0)^2)^{1/2} \\ = 5$$

$$x \rightarrow 1$$

$$f(x) = (25 - (1)^2)^{1/2} \\ = 4.89$$

$$x = 2$$

$$f(x) = (25 - (2)^2)^{1/2} \\ = 4$$

$$x \rightarrow 3$$

$$f(x) = (25 - (3)^2)^{1/2} \\ = 4$$

$$x \rightarrow 4$$

$$f(x) = (25 - (4)^2)^{1/2} \\ = 3$$

$$x \rightarrow 5$$

$$f(x) = (25 - (5)^2)^{1/2} \\ = 0$$

The function on the interval $[-5, 5]$ is continuous.