When $t = 0$, $dx/dt = 0$

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \tfrac{1}{10}(\sin(0) - \cos(0))$$

$$0 = -2A - 3B + \tfrac{1}{10}$$

$$2A + 3B = 0.1 \quad \text{———— (ii)}$$

from eq (i)

$$A = -B$$

$$2(-B) + 3B = 0.1$$

$$-2B + 3B = 0.1$$

$$B = 0.1$$

$$B = \tfrac{1}{10}$$

$$A = -0.1$$

$$A = -\tfrac{1}{10}$$

$$\therefore\ x = -\tfrac{1}{10}e^{-2t} + \tfrac{1}{10}e^{-3t} + \tfrac{1}{10}(\cos t + \sin t)$$

$$x = \tfrac{1}{10}(e^{-3t} - e^{-2t} + \cos t + \sin t)$$

b) commandwindow

```
clc
clc
clear all
syms t
t=0:0.01:15
x=(-(1/10)*exp(-(2*t)))+(1/10*exp(-(3*t)))+1/10*(cos(t)+
  sin(t))
plot(t,x)
grid on
grid minor
axis tight
ylabel('vibrations')
xlabel('time')
```

c $x_{ss} = \tfrac{1}{10}\cos t + \tfrac{1}{10}\sin t$

$$\tfrac{1}{10}\cos t + 0.1\sin t = k\sin(t + a)$$

$$k\sin(t + a) = k\sin t\cos a + k\cos t\sin a$$

coefficient of cos t

$$0.1 = k\sin a$$

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ENG 381 Assignment 1

The dynamic model of a body in motion performing damped force vibrations is as in eq (i)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t \quad \text{——— (i)}$$

Given that when $t = 0$, $x = 0$ and $dx/dt = 0$

a. Using the auxiliary equation method, obtain the solution of the model in form of an expression having x as a function of t.

b. With the aid of a MATLAB agile program, plot the relationship between x and t for $0 \leq t \leq 15$ time unit using a step size of 0.01 unit, and

c. Write the steady-state solution of the model in form of $x = K\sin(t + \alpha)$

Solution

a. $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$

$m^2 + 5m + 6 = 0$

$m^2 + 3m + 2m + 6 = 0$

$m(m+3) + 2(m+3) = 0$

$(m+2)(m+3) = 0$

$m + 2 = 0 \qquad m + 3 = 0$

$m_1 = -2, \quad m_2 = -3$

C.F => $x = Ae^{-2t} + Be^{-3t}$

P.I => $f(t) = \cos t$;

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

coefficient of sin t

$0.1 = k\cos\alpha$

squaring both sides we get

$k^2\sin^2\alpha + k^2\cos^2\alpha = 0.01 + 0.01$

$k^2(\sin^2\alpha + \cos^2\alpha) = 0.01 + 0.01$

$k^2 = 0.02$

$k = \sqrt{0.02}$

$k = 0.1414$

$\frac{k\sin\alpha}{k\cos\alpha} = \frac{0.1}{0.1}$

$\frac{\sin\alpha}{\cos\alpha} = 1$

$\tan\alpha = 1$

$\alpha = \tan^{-1}(1)$

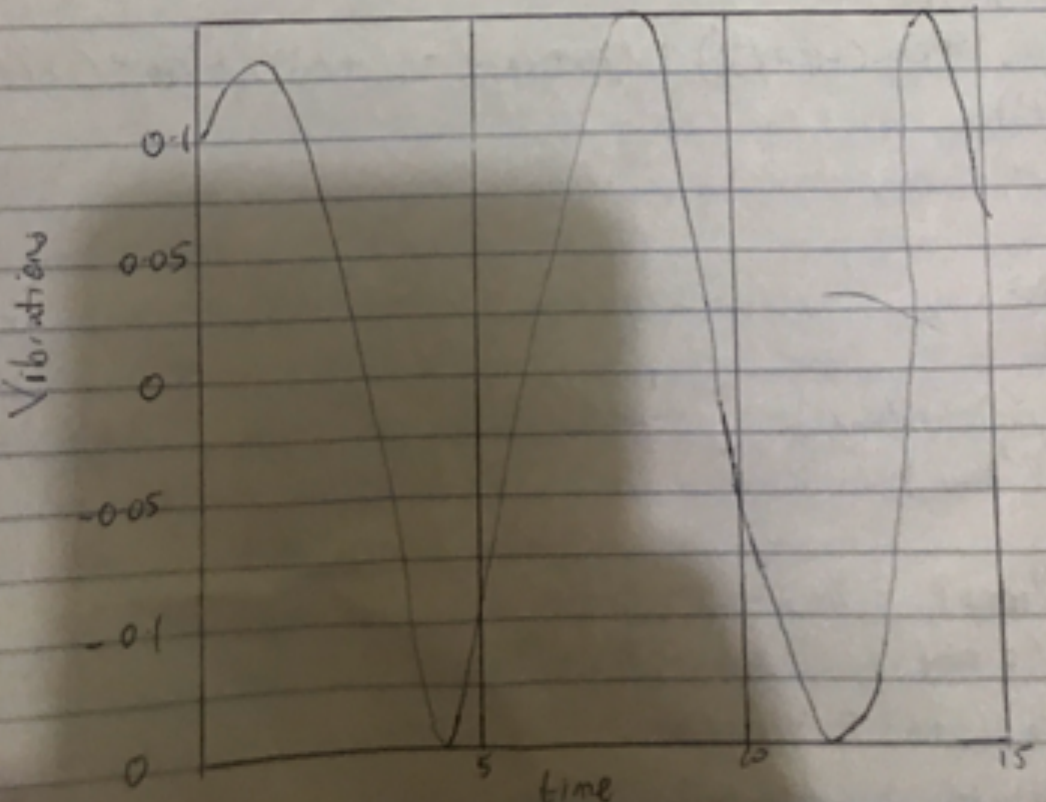
$\alpha = 45°$

$\alpha = \pi/4$

∴ steady state for the equation

$x_{ss} = \frac{\sqrt{2}}{10}\sin(t + \pi/4)$

b(ii)



$(-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$-C\cos t + 5D\cos t + 6C\cos t - D\sin t - 5C\sin t + 6D\sin t = \cos t$

$\cos t(-C + 5D + 6C) + \sin t(-D - 5C + 6D) = \cos t$

$5C + 5D = 1$ ——— (i)

$-5C + 5D = 0$ ——— (ii)

$5D = 1 - 5C$ ——— (iii)

$-5C + 1 - 5C = 0$

$1 - 10C = 0$

$1 = 10C$

$C = 1/10$

putting C into eq (iii)

$5D = 1 - 5(1/10)$

$5D = 1 - 1/2$

$5D = 1/2$

$D = 1/2 \times 1/5 = 1/10$

P.I, $x = 1/10\cos t + 1/10\sin t$

G.S = C.F + P.I

$x = Ae^{-2t} + Be^{-3t} + 1/10\cos t + 1/10\sin t$

When $t = 0$, $x = 0.1$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 1/10\cos(0) + 1/10\sin(0)$

$0.1 = A + B + 1/10$

$A + B = 0.1 - 1/10$

$A + B = 0$ ——— (i)

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - 1/10\sin t + 1/10\cos t$

$\frac{dx}{dt} = -2Ae^{-2t} + 3Be^{-3t} - 1/10(\sin t - \cos t)$