

c) steady state equation

$$x_{ss} = Y_0 \cos \omega t + Y_0 \sin \omega t$$

$$Y_0 \cos \omega t + 0.1 \sin \omega t = K \sin(\omega t + \alpha)$$

$$K \sin(\omega t + \alpha) = K \sin \omega t \cos \alpha + K \cos \omega t \sin \alpha$$

→ coeff of  $\cos \omega t$

$$0.1 = K \cos \alpha$$

→ coeff of  $\sin \omega t$

$$K = K \sin \alpha$$

Squaring both sides

$$K^2 \sin^2 \alpha + K^2 \cos^2 \alpha = 0.01 + 0.01$$

$$K^2 (\sin^2 \alpha + \cos^2 \alpha) = 0.01 + 0.01$$

$$K^2 = 0.02$$

$$K = \sqrt{0.02}$$

$$K = 0.1414 \dots$$

Phase

$$\frac{K \sin \alpha}{K \cos \alpha} = \frac{0.1}{0.1}$$

$$\frac{\sin \alpha}{\cos \alpha} = 1$$

$$\cos \alpha$$

$$\tan \alpha = 1$$

$$\alpha = \tan^{-1}(1)$$

$$\alpha = 45^\circ$$

$$\alpha = \pi/4$$

Steady state

$$x_{ss} = \frac{\sqrt{2}}{15} \sin\left(t + \frac{\pi}{4}\right)$$

## 2. MATLAB CODES

command window

clc

clear

close all

syms t

$$t = 0:0.01:15$$

$$Scout + 50 \cos t = \cos t$$

$$SD \sin t - SC \sin t = 0$$

$$\therefore SC + SD = 1$$

$$SD - SC = 0$$

$$\therefore SD = SC$$

$$\therefore SC + SC = 1$$

$$2SC = 1$$

$$SC = \frac{1}{2}$$

$$\text{then } SD = \frac{SC}{10}$$

$$SD = \frac{1}{2}$$

$$D = \frac{1}{100}$$

$$PI = \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$X = Ae^{-2t} + Be^{-3t} + \frac{1}{10} (\cos t + \sin t)$$

$$\text{Sub } X = 0 \text{ at } t = 0$$

$$0 = A e^{-2(0)} + B e^{-3(0)} + \frac{1}{10} (\cos 0 + \sin 0)$$

$$0 = A + B + \frac{1}{10}$$

$$A + B = 0 - \frac{1}{10}$$

$$A + B = 0 \quad \text{--- (1)}$$

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} (\cos t + \sin t)$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + \frac{1}{10} (-\sin t + \cos t)$$

$$0 = -2Ae^{-2(0)} - 3B e^{-3(0)} + \frac{1}{10} (-\sin 0 + \cos 0)$$

$$0 = -2A - 3B + \frac{1}{10}$$

$$-\frac{1}{10} = -2A - 3B$$

$$\frac{1}{10} = 2A + 3B \quad \text{--- (2)}$$

since  $A + B = 0$

$$A = -B$$

$$\frac{1}{10} = -B + 3B$$

$$\frac{1}{10} = 2B$$

$$\text{then } B = -\frac{1}{20}$$

Sub A and B in eqn given eqn

$$x = \left[ -\frac{1}{20} e^{-2t} + \frac{1}{10} e^{-3t} + \frac{1}{10} (\cos t + \sin t) \right]$$

- i. The dynamic model of a body in performing damped forced vibration is as in Equation (i)
- $$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$
- Given that when  $t=0$ ,  $x=0.1$  and  $\frac{dx}{dt}=0$
- a. Using the Auxiliary Equation method, obtain the solution of the model in form of an expression having  $x$  as a function of  $t$ .
- b. With the aid of a MATLAB script program, plot the relationship between  $x$  and  $t$  for  $0 \leq t \leq 15$  time unit using a step size of 0.01 unit, and
- c. Write the steady state solution of the model in form of  $x = k \sin(t + \alpha)$

SOLUTION

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$X = C.F + P.I$$

$$m^2 + 5m + 6 = 0 \rightarrow CP$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$(m+2)(m+3) = 0$$

$$m = -2 \text{ or } -3$$

$$C.F = Ae^{-2t} + Be^{-3t}$$

$$P.I = C \cos t + D \sin t : \#$$

$$\frac{dx}{dt} = C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

then :

$$-C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t + D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$5D \sin t - 5C \sin t + 5C \cos t + 5D \cos t = \cos t$$