

$$5D - 5C = 0$$

$$5D = 5C$$

$$D = \frac{5C}{5}$$

$$D = C$$

$$5C + 5D = 0$$

$$5C = -5D$$

$$C = \frac{-5D}{5}$$

$$\therefore C = -5D = \frac{40}{5} = 8$$

Solving simultaneously

$$5C + 5D = 1$$

$$-5C + 5D = 0$$

$$100 = 1 \quad D = \frac{1}{10}$$

$$\text{Substitution: } 5C + 5(\frac{1}{10}) = 1$$

$$5C + \frac{1}{2} = 1$$

$$5C = 1 - \frac{1}{2}$$

$$C = \frac{1}{2} = \frac{1}{2} \times \frac{1}{5} \quad \therefore C = \frac{1}{10}$$

$$\text{General Solution}(D=0) = Ae^{-3t} + Be^{-3t} + \frac{1}{10}(\text{cont} + \text{cost})$$

$$\text{when } t=0, D=0+1$$

$$0+1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}(\text{cont} + \text{cost})$$

$$0+1 = A + B + \frac{1}{10}(\text{cont} + \text{cost})$$

$$A + B = 0+1 \quad \text{--- (1)}$$

$$\text{when } t=0, \frac{dD}{dt} = 0$$

$$\frac{dD}{dt} = -2Ae^{-2t} - 3Be^{-3t} + 0+1(\text{cost} - \text{cont})$$

$$0 = -2Ae^{0+0} - 3Be^{0+0} + 0+1(C\cos 0 - S\sin 0)$$

$$0 = -2A - 3B + 0+1$$

$$-0+1 = -2A + 3B$$

$$\text{Recall } A + B = 0+1. \quad \text{To find } B$$

$$A = -B$$

$$-0+1 = -2(-B) - 3B$$

$$-0+1 = +2B - 3B$$

$$-0+1 = -B$$

$$B = 0+1$$

CHEMICAL ENGINEERING

ENG 281

Solutions

1. The dynamic model of a body in motion performing damped forced vibrations is as in eq(1)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that when $t=0$, $x=0.1$ and $dx/dt=0$.

- ② Using the auxiliary Equation method, obtain the solution of the model in form of an expression having x as a function of t .

Assuming $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$

$$m^2 + 5m + 6 = 0$$

$$m^2 + 6m + 2m + 6 = 0$$

$$(m+3)(m+2) = 0$$

$$m_1 = -3 \quad m_2 = -2$$

$$x = Ae^{-3t} + Be^{-2t}$$

$$P_1 = \cos t$$

$$x = C\cos t + D\sin t$$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

Substituting into the equation

$$(-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$$

$$-C\cos t - D\sin t + 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$$

$$-C\cos t - D\sin t + 6C\cos t + 6D\sin t - 5C\sin t + 5D\cos t = \cos t$$

$$(-C\cos t + 6C\cos t) - (D\sin t + 6D\sin t) - 5C\sin t + 5D\cos t = \cos t$$

$$5C\cos t + 5D\sin t - 5C\sin t + 5D\cos t = \cos t$$

$$5C\cos t + 5D\cos t + 5D\sin t - 5C\sin t = \cos t$$

$$\cos t(5C + 5D) + \sin t(5D - 5C) = \cos t$$

$$5C + 5D = 1 \text{ and } 5D - 5C = 0$$

To find A

$$A = -B$$

$$A = -0.1$$

$$z. \eta_c = -0.1e^{-3t} + 0.1e^{3t} + \frac{1}{10}[3\sin t + \cos 2t] \quad \text{or}$$

$$\eta_c = \frac{-1}{10}e^{-3t} + \frac{1}{10}e^{3t} + \frac{1}{10}[3\sin t + \cos 2t]$$

2. Command window

clear

clc

close all

symt

$$\eta_c = (1/10 * \exp(-2*t)) - (1/10 * \exp(3*t)) + (1/10(\sin(t) + \cos(t)))$$

$$t = 0:0.01:15$$

$$\eta_{ct} = \text{subs}(\eta_c, t)$$

$$\eta_{ctn} = \text{double}(\eta_{ct})$$

$$\text{Plot}(t, \eta_{ctn})$$

$$\text{xlabel('t')}$$

$$\text{y_label('x')}$$

grid on

grid minor

grid right.

3. At steady state:

$$\eta_c = \eta_c = 0.1\cos t + 0.1\sin t$$

$t \rightarrow \infty$ steady state

$$0.1\cos t + 0.1\sin t = k_{a0}(t+\alpha)$$

$$k_{a0}(t+\alpha) = k_{a0}\cos t + k_{a0}\sin t$$

$$\text{coefficient of } \cos t = k_{a0}\cos t$$

$$\text{coefficient of } \sin t = k_{a0}\sin t$$

When squaring both sides,

$$k^2 \sin^2 \alpha + k^2 \cos^2 \alpha = 0.1^2 + 0.1^2$$

$$k^2 (\sin^2 \alpha + \cos^2 \alpha) = 0.02$$

$$K^2 = 0.02$$

$$K = \sqrt{0.02}$$

$$K = 0.141 \approx \sqrt{2/10}$$

$$\frac{K \sin \alpha}{K \cos \alpha} = \frac{0.1}{0.1} = 1$$

Remember that $\sin/\cos = \tan$

$$\tan \alpha = 1$$

$$\tan^{-1}(1) = \alpha$$

$$\alpha = 45^\circ \text{ or } \pi/4 \text{ radian}$$

Steady state

$$= \sqrt{2} \sin \left(t + \pi/4 \right)$$

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