Name: OFOEGBU CHIJIOKE FORTUNE

Department: COMPUTER ENGINEERING

Matric No: 17/ENG02/062

Assignment

1) The dynamic model of a body in motion performing damped forced vibration is an equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \text{Cost}$$

Given that when $t=0$, $x=0.1$ and $dx/dt = 0$

(a) Using the auxiliary equation obtained the solution of the model in form of an expression having c as a function of t.

(b) With aid of a MATLAB MFILE program plot the relationship between x and t for $0 \le t \le 15$ time unit using a step size of 0.01 unit

(c) With the study state solution of the model in form of $x = k\sin(t+\phi)$

Solution

(a) $$\frac{d^2x}{dt^2} + \frac{5dx}{dt} = 6x = \text{cost}$$

Auxillary Equation; $m^2 + 5m + 6 = 0$

$$m^2 + 2m + 3m + 6 = 0$$

$$m(m+2) + 3(m+2) = 0$$

$$(m+3)(m+2) = 0$$

$m = -3$ or -2

Complementry function $x = Ae^{-3x} + Be^{-2x}$

Particular Integral: Assume $x = C\cos t + D\sin t$

$$\therefore \frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = C\cos - D\sin t$$

Putting the value of d^2x/dt^2 and dx/dt in the equation

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

Comparing coefficient

Coefficient of $\cos t$: $C + 5D + 6C = 1$ ..... (1)

coefficient of $\sin t$: $-D - 5C + 6D = 0$ ..... (2)

from eqn (1) $5C + 5D = 1$ ..... (3)

from eqn (2) $5D - 5C = 0$ ..... (4)

from eqn (1) $5D = 1 - 5C$ ..... (5)

Put equation (5) in equation (4)

$1 - 5c - 5c = 0$

$1 - 10c = 0$

$C = 1/10;\ 0.1$

$\therefore 5D = 1 - 5c$

Put the value of C in eqn (5)

$5D = 0.5$

```
xn = subs(x
Plot(t, xn)
xlabel('time')
grid on
grid minor
axis
```

- MATLAB

```
Command window
clc
clear all
close all
syms x,t
t=0:0.01:15
x=0.1*exp(-3*t)-0.1*exp(-2*t)+0.1*cos(t)+0.1*sin(2t)
xn= subs(x)
Plot(t,xn)
xlabel('Time')
grid on
grid minor
axis tight
```

$D = \frac{0.5}{5} = 0.1$

Particular solution: $x = Ae^{-3t} + Be^{-2t} + D\sin t + 0.1\cos t$

When $t=0$; $x = 0.1$ $dx/dt = 0$

$0.1 = A + B + 0.1$

$A + B = 0$ ..... (6)

$dy/dx = -3Ae^{-3t} - 2Be^{-2t} + 0.1\sin t + 0.1\cos t$

$0 = -3A - 2B + 0.1$

from equ (6) $B = -A$

$0 = -3A - 2(-A) + 0.1$

$0 = -A + 0.1$

and $B = -A$

$B = 0.1$

General solution $= 0.1e^{-3t} - 0.1e^{-2t} + 0.1\sin t + 0.1\cos t$

2 Command Window

```
clear
clc
clear all
Syms x t
x=0.1*exp(-3*t) - 0.1* exp(-2*t)+0.1*cos(t)+0.1*sin t
t= 0:0.1:15
```

(d) $0.1\cos t + 0.1\sin t = k\sin(t+\alpha)$ at steady state

$0.1\cos t + 0.1\sin t = k\sin t\cos\alpha + k\cos t\sin\alpha$

Comparing co-efficient

Coefficient of $\cos t$

$0.1 = k\sin\alpha$

Co-efficient of $\sin t$

$0.1 = k\cos\alpha$

Assume

$k^2\sin^2\alpha + k^2\cos^2\alpha = 0.1^2 + 0.1^2$

$k^2(\sin^2\alpha + \cos^2\alpha) = 0.02$

$k^2 = 0.02 \quad \ldots (\sin^2\alpha + \cos^2\alpha) = 1$

$\therefore k^2 = \frac{2}{100}$

$k = \sqrt{\frac{2}{100}}$

$\frac{k\sin\alpha}{k\cos\alpha} = \frac{0.1}{0.1} = 1$

$\tan\alpha = 1$

$\alpha = \tan^{-1}(1)$

$\alpha = \pi/4$ or 45°

x at steady state; $\frac{\sqrt{2}}{10}\sin\left[\frac{\pi}{4} + t\right]$.

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