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CIVIL ENGINEERING.
17/ENG03/053
ENG 381

① $\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$ at $t=0, x=0.1, \frac{dx}{dt}=0$

solution

$m^2+5m+6=0 \longrightarrow$ Assuming $f(x)=0$

$m^2+2m+3m+6=0$

$m(m+2)+3(m+2)=0$

$(m+2)(m+3)=0$

$m_1=-2$ & $m_2=-3$

$C.F = x = Ae^{mx}x + Be^{mx}x$

$x = Ae^{-2t} + Be^{-3t}$

$P.I = f(x) = \cos t$

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$.

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$= -(C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t)$

$= \cos t$

$= -C\cos t + 5D\cos t + 6C\cos t - D\sin t - 5C\sin t + 6D\sin t = \cos t$.

$= (-C+5D+6C)\cos t + (-5C-D+6D)\sin t$ ~~$+6D\sin t$~~ $= \cos t$.

$(5C+5D)\cos t + (5C+5D)\sin t = \cos t + D\sin t$.

$\longrightarrow 5C+5D=1$ ——— ①

$-5C+5D=0$ ——— ②

Solving simultaneously

$10D = 1$

$\rightarrow D = \frac{1}{10}$

Sub $D = \frac{1}{10}$ into equ 1.

$5C + 5\left(\frac{1}{10}\right) = 1$

$5C + \frac{1}{2} = 1$

$5C = \frac{1}{2}$

$= 10C = 1$

$C = \frac{1}{10}$

$P.I = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$G.S = \frac{1}{10}(\cos t + \sin t) + Ae^{-2x} + Be^{-3x}$

$x = \frac{1}{10}(\cos t + \sin t) + Ae^{-2t} + Be^{-3t}$

$0.1 = \frac{1}{10}(\cos t + \sin t) + Ae^{0} + Be^{0}$

$0.1 = \frac{1}{10}(1 + 0) + A + B$

$0.1 = \frac{1}{10} + A + B$

$\rightarrow A + B = 0$ ——— ①

$x = \frac{1}{10}(\cos t + \sin t) + Ae^{-2t} + Be^{-2t}$

$\frac{dx}{dt} = \frac{1}{10}(-\sin t + \cos t) - 2Ae^{-2t} - 3Be^{-3t}$

$0 = \frac{1}{10} - 2A - 3B$

$-2A - 3B = -\frac{1}{10}$ ——— ②

$A + B = 0$ ——— ①

$-2A - 3B = -\frac{1}{10}$ ——— ②

$2A + 2B = 0$

$-2A - 3B = -\frac{1}{10}$

$-B = -\frac{1}{10}$

$B = \frac{1}{10}$

But $B = \frac{1}{10}$ into equ I

$A + \frac{1}{10} = 0$

$A = -\frac{1}{10}$

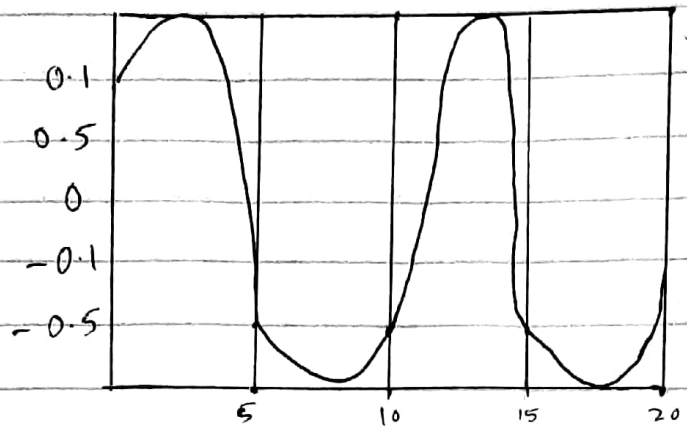
$x = \frac{1}{10}\cos t + \frac{1}{10}\sin t - \frac{1}{10}e^{-2t} + \frac{1}{10}e^{-3t}$

$x = \frac{1}{10}(\cos t + \sin t - e^{-2t} + e^{-3t}$

(b) MATLAB

Command window

```
clear
clc
Clear all
sign t
t = 0:0.01:15
x = 0.1*(exp(-3*t) -Exp(-2*t) + (os(t) + Sin(t))
xn = subs(x)
    plot (t, xn)
    axis tight
    grid on
    grid minor
```



$x = k\sin(t+0)$

knowing that $x = 0.1$, $t = 0$ & $\frac{dx}{dt} = 0$,

$0 = k\cos(0+a)$

$k\cos(a) = 0$

$0.1 = k\sin(0+a)$

$k\sin(a) = 0.1$ ..... (1)

$\cos a = 0$

$a = \cos^{-1} = 0$

$= 40^{\circ}$

Sub a into (1)

$0.1 = k\sin(90)$

$\therefore k = \frac{0.1}{\sin 90} = 0.1$

$x = 0.1(\sin(t+90))$

command window

```
close all
clear
clc
Syms t, x
t = (0:0.01:15)
x = 0.1*(sin(t+90))
plot(t, x)
```

Question 3

At steady state

$x_{ss} = x_{steady\ state} = 0.1\cos t + 0.1\sin t$

$0.1\cos t + 0.1\sin t = K\sin(t+\alpha)$

$K\sin(t+\alpha) = K\sin t\cos\alpha + K\cos t\sin\alpha$

NB coefficient of $\cos t = K\sin\alpha$

NB coefficient of $\cos t = K\cos\alpha$

When squaring both sides

$K^2\sin^2\alpha + K^2\cos^2\alpha = 0.1^2 + 0.1^2$

$K^2(\sin^2\alpha + \cos^2\alpha) = 0.02$

$K^2 = 0.02$

$K = \sqrt{0.02}$

$K = 0.144 = \sqrt{2}/10$

$\frac{K\sin\alpha}{K\cos\alpha} = \frac{0.1}{0.1} = 1$

Remember that $\sin/\cos = \tan$

$\tan\alpha = 1$

$\tan^{-1}(1) = \alpha$

$\alpha = 45^\circ$ or $\pi/4$ radian

Steady state

$= \frac{\sqrt{2}}{40}\sin(t + \pi/4)$