

Assignment 1

K&OIN JOHN MICHAEL AGIM
17/EUG05/004

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

C.F. $m^2 + 5m + 6 = 0$

Using Completing the square method

$$m^2 + 5m + \left(\frac{-5}{2}\right)^2 = -6 + \frac{25}{4}$$

$$\left(m + \frac{5}{2}\right)^2 = \frac{1}{4} \quad \text{Using root both sides}$$

$$m + \frac{5}{2} = \pm \frac{1}{2}$$

$$m = -3 \quad \text{or} \quad m = -2$$

$$x = C.F. = Ae^{-3t} + Be^{-2t}$$

Particular Integral

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$(-C \cos t - D \sin t) + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$5C \cos t + 5D \cos t = \cos t \quad \dots \text{eqn (1)}$$

$$-5C \sin t + 5D \sin t = 0 \quad \dots \text{eqn (2)}$$

Simultaneous equation

$$5C + 5D = 1$$

$$-5C + 5D = 0$$

$$10D = 1$$

$$D = \frac{1}{10}$$

$$5C + 5\left(\frac{1}{10}\right) = 1$$

$$C = -\frac{1}{2} * \frac{1}{5}$$

$$C = -\frac{1}{10}$$

$$P.I. = x = -\frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$x = Ae^{-3t} + Be^{-2t} + \frac{1}{10}(\sin t - \cos t)$$

where $t=0$, $x=0.1$ and $\frac{dx}{dt}=0$

$$0.1 = A(1) + B(1) + \frac{1}{10}(-1)$$

$$0.1 = A + B - 0.1$$

$$A + B = 0.1 + 0.1$$

$$A + B = 0.2 \dots \dots \text{eqn (3)}$$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} + \frac{1}{10}(\cos t + \sin t)$$

$$0 = -3A - 2B + \frac{1}{10}(1)$$

$$-0.1 = 3A - 2B$$

$$3A + 2B = 0.1 \dots \dots \text{eqn (4)}$$

Thus,

$$A + B = 0.2$$

$$A = 0.2 - B \dots \dots \text{eqn (5)}$$

Sub eqn (5) into eqn (4)

$$3(0.2 - B) + 2B = 0.1$$

$$0.6 - 3B + 2B = 0.1$$

$$-B = -0.5$$

$$B = 0.5$$

$$A = -0.3$$

Therefore

$$x = -0.3e^{-3t} + 0.1e^{-2t} + \frac{1}{10}(\sin t - \cos t)$$
$$= 0.1e^{-2t} - 0.3e^{-3t} + \frac{1}{10}(\sin t - \cos t)$$

Command window:

close all;

clear all;

clc;

Sym x, t

t = [0:0.1:15];

x = 0.1 * exp(-2*t) - 0.3 * exp(-3*t) + 0.1 * (sin(t) - cos(t));

Plot(t, x)

Question 11c)

$$K = K_0 \cos(t + \alpha)$$

Assuming $x = 0.1$ at $t = 0$ & $\frac{dx}{dt} = 0$

$$\frac{dx}{dt} = K_0 \sin(t + \alpha)$$

$$0 = K_0 \cos(0 + \alpha)$$

$$K_0 \cos(\alpha) = 0$$

$$0 = K_0 \sin(0 + \alpha) \dots \dots \dots \sin(\alpha)$$

$$K_0 \sin(\alpha) = 0.1$$

$$\cos(\alpha) = 0 \quad \alpha = \cos^{-1}(0)$$

$$\alpha = 90^\circ$$

Subst. into eqn (1)

$$0.1 = k \sin(90)$$

$$k = \frac{0.1}{\sin 90}$$

$$x = 0.1 (\sin(t + 90))$$

Command window

close all;

clear all;

clc;

sym t, x

t = [0:0.1:15]';

x = 0.1 * (sin(t + 90));

plot(t, x)