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COURSE: ENG 281 - ENGINEERING MATHS

DEPT: COMPUTER ENG.

(a) Show that the limit of the function given in Equation (1.1) as n approaches 0 is $\frac{a}{b}$.

$$f(n) = \frac{\sin a(n)}{bn}$$

Sol:

Direct substitution

$$\lim_{n \rightarrow 0} \frac{\sin a(n)}{bn} = \frac{\sin a(0)}{b(0)}$$

$$= \frac{0}{0} = \text{Indeterminate}$$

Using L'Hopital's rule

$$\begin{aligned} \lim_{n \rightarrow 0} \frac{\sin a(n)}{bn} &= \frac{a \cos a(n)}{b} = \frac{a \cos a(0)}{b} \\ &= \frac{a \cos 0}{b} = \frac{a}{b} \end{aligned}$$

(b) The model of a system has been developed to be given in equation (1.2)

$$f(n) = 5n - 21$$

given that $\delta = 0.1$ and $\Delta \delta = 0.01$, demonstrate, in tabular form, that the limit of the model as $n \rightarrow 6$ is equal to 9.

Sol:

$$\begin{aligned} \delta = 0.1 & \quad \Delta \delta = 0.01 & \quad 6 - 0.1 = 5.9 \\ \epsilon = 0.5 & \quad \Delta \epsilon = 0.05 & \quad 9 - 0.5 = 8.5 \end{aligned}$$

$0.01 + n$	$0.05 + \epsilon$	$n - 0.01$	$n - 0.05$
5.9	8.5	6.1	9.50
5.91	8.55	6.09	9.45
5.92	8.60	6.08	9.40
5.93	8.65	6.07	9.35
5.94	8.70	6.06	9.30
5.95	8.75	6.05	9.25
5.96	8.80	6.04	9.20
5.97	8.85	6.03	9.15
5.98	8.90	6.02	9.10
5.99	8.95	6.01	9.05
6	9	6	9

(c) Show whether the function in Equation (1.3) is continuous on the interval $(-5, 5)$

$$f(n) = (25 - n^2)^{1/2}$$

sol

For -5:

$$\begin{aligned} \lim_{n \rightarrow -5} (25 - n^2)^{1/2} &= (25 - (-5)^2)^{1/2} \\ &= (25 - 25)^{1/2} \\ &= 0 \end{aligned}$$

For -4

$$\begin{aligned} \lim_{n \rightarrow -4} (25 - n^2)^{1/2} &= (25 - (-4)^2)^{1/2} \\ &= (25 - 16)^{1/2} \\ &= \sqrt{9} = 3 \end{aligned}$$

For -3

$$\begin{aligned} \lim_{n \rightarrow -3} (25 - n^2)^{1/2} &= (25 - (-3)^2)^{1/2} \\ &= (25 - 9)^{1/2} \\ &= \sqrt{16} = 4 \end{aligned}$$

For -2

$$\begin{aligned} \lim_{n \rightarrow -2} (25 - n^2)^{1/2} &= (25 - (-2)^2)^{1/2} \\ &= (25 - 4)^{1/2} \\ &= \sqrt{21} = 4.58 \end{aligned}$$

For -1

$$\lim_{n \rightarrow -1} (25 - n^2)^{\frac{1}{2}} = (25 - (-1)^2)^{\frac{1}{2}}$$

$$= (25 - 1)^{\frac{1}{2}}$$

$$= \sqrt{24} = 4.89$$

For 0

$$\lim_{n \rightarrow 0} (25 - n^2)^{\frac{1}{2}} = (25 - (0)^2)^{\frac{1}{2}}$$

$$= (25 - 0)^{\frac{1}{2}}$$

$$= \sqrt{25} = 5$$

For 1

$$\lim_{n \rightarrow 1} (25 - n^2)^{\frac{1}{2}} = (25 - (1)^2)^{\frac{1}{2}}$$

$$= (25 - 1)^{\frac{1}{2}}$$

$$= \sqrt{24} = 4.89$$

For 2

$$\lim_{n \rightarrow 2} (25 - n^2)^{\frac{1}{2}} = (25 - (2)^2)^{\frac{1}{2}} = 4.58$$

For 3

$$\lim_{n \rightarrow 3} (25 - n^2)^{\frac{1}{2}} = (25 - (3)^2)^{\frac{1}{2}} = 4$$

For 4

$$\lim_{n \rightarrow 4} (25 - n^2)^{\frac{1}{2}} = (25 - (4)^2)^{\frac{1}{2}} = 3$$

For 5

$$\lim_{n \rightarrow 5} (25 - n^2)^{\frac{1}{2}} = (25 - (5)^2)^{\frac{1}{2}} = 0$$

$\therefore$ the ~~inter~~ function $f(n) = (25 - n^2)^{\frac{1}{2}}$ is continuous on the interval $(-5, 5)$