

CHINEDUM PRUDENCE ESE
 CHEMICAL ENGINEERING
 HENG01/009
 ENG 381

$$D \frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 6x = \cos t \text{ at } t=0 \quad x=0, \frac{dx}{dt} = 0$$

Solution

$$m^2 + 5m + 6 = 0 \rightarrow \text{Assuming } f(x) = 0$$

$$m^2 + 2m + 3m + 6 = 0$$

$$m(m+2) + 3(m+2) = 0$$

$$(m+2)(m+3) = 0$$

$$m_1 = -2 \cdot \text{and } m_2 = -3$$

$$C.F = x = A e^{m_1 x} + B e^{m_2 x}$$

$$x = A e^{-2t} + B e^{-3t}$$

$$P.I = f(x) = \cos t$$

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$-C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$= -C \cos t + 5D \cos t + 6C \cos t - D \sin t - 5C \sin t + 6D \sin t = \cos t$$

$$= (5C + 5D) \cos t + (-5C - D + 6D) \sin t = \cos t$$

$$(5C + 5D) \cos t + (5C + 5D) \sin t = \cos t + D \sin t$$

$$\rightarrow 5C + 5D = 1 \quad \text{--- (1)}$$

$$-5C + 5D = 0 \quad \text{--- (2)}$$

Solving simultaneously

$$10D = 1$$

$$\rightarrow D = \frac{1}{10}$$

Sub $D = \frac{1}{10}$ into eqn 1.

$$5C + 5\left(\frac{1}{10}\right) = 1$$

$$5C + \frac{1}{2} = 1$$

$$5C = \frac{1}{2}$$

$$= 10C = 1$$

$$C = \frac{1}{10}$$

$$P.I = \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$C.F.S = \frac{1}{10} (\cos t + \sin t) + Ae^{-2t} + Be^{-3t}$$

$$x = \frac{1}{10} (\cos t + \sin t) + Ae^{-2t} + Be^{-3t}$$

$$0.1 = \frac{1}{10} (\cos t + \sin t) + Ae^0 + Be^0$$

e

$$0.1 = \frac{1}{10} (1 + 0) + A + B$$

$$0.1 = \frac{1}{10} + A + B$$

$$\rightarrow A + B = 0 \quad \text{--- (1)}$$

$$x = \frac{1}{10} (\cos t + \sin t) + Ae^{-2t} + Be^{-3t}$$

$$\frac{dx}{dt} = \frac{1}{10} (-\sin t + \cos t) - 2Ae^{-2t} - 3Be^{-3t}$$

$$0 = \frac{1}{10} - 2A - 3B$$

$$-2A - 3B = \frac{1}{10} \quad \text{--- (2)}$$

$$A + B = 0 \quad \text{--- (1)}$$

$$-2A - 3B = -\frac{1}{10} \quad \text{--- (2)}$$

$$2A + 2B = 0$$

$$-2A - 3B = -\frac{1}{10}$$

$$-B = -\frac{1}{10}$$

$$B = \frac{1}{10}$$

Put $B = \frac{1}{10}$ into eqn 1

$$A + \frac{1}{10} = 0$$

$$A = -\frac{1}{10}$$

$$x = \frac{1}{10} \cos t + \frac{1}{10} \sin t - \frac{1}{10} e^{-2t} + \frac{1}{10} e^{-3t}$$

$$x = \frac{1}{10} (\cos t + \sin t - e^{-2t} + e^{-3t})$$

b) MATLAB

Command window

clear

clc

clear all

signs t

t = 0:0.01:15

x = 0.1 * (exp(-3*t) - exp(-2*t) + cos(t) + sin(t))

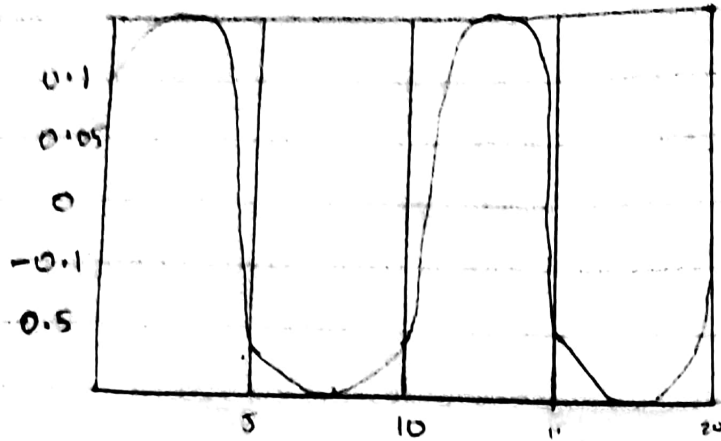
xn = subs(x)

plot(t, xn)

axis tight

grid on

grid minor



$$x = K \sin(\omega t + \phi)$$

Knowing that $x = 0.1$, $t = 0$ & $\frac{dx}{dt} = 0$

$$0 = K \cos(\omega t + \phi)$$

$$\therefore K \cos(\phi) = 0$$

$$0.1 = K \sin(\omega t + \phi)$$

$$K \sin(\phi) = 0.1 \dots \textcircled{1}$$

$$\cos \phi = 0$$

$$\phi = \cos^{-1} 0$$

$$= 90^\circ$$

Sub ϕ into $\textcircled{1}$

$$0.1 = K \sin(90)$$

$$\therefore K = \frac{0.1}{\sin 90} = 0.1$$

$$x = 0.1 (\sin(\omega t + 90))$$

Command window

close all

clear

clc

Syms t, x

$$t = (0:0.01:15)$$

$$x = 0.1 * (\sin(t + 90))$$

plot (t,x)

Question 3

At steady state

$$x_{ss} = \text{steady state} = 0.1 \cos t + 0.1 \sin t$$

$$0.1 \cos t + 0.1 \sin t = K \sin(t + \alpha)$$

$$K \sin(t + \alpha) = K \sin t \cos \alpha + K \cos t \sin \alpha$$

$$\text{NB coefficient of } \cos t = K \sin \alpha$$

$$\text{NB coefficient of } \sin t = K \cos \alpha$$

When squaring both sides

$$K^2 \sin^2 \alpha + K^2 \cos^2 \alpha = 0.1^2 + 0.1^2$$

$$K^2 (\sin^2 \alpha + \cos^2 \alpha) = 0.02$$

$$K^2 = 0.02$$

$$K = \sqrt{0.02}$$

$$K = 0.144 = \frac{\sqrt{2}}{10}$$

$$\frac{K \sin \alpha}{0.1} = 1$$

$$\frac{K \cos \alpha}{0.1} = 1$$

Remember that $\sin / \cos = \tan$

$$\tan \alpha = 1$$

$$\tan^{-1}(1) = \alpha$$

$$\alpha = 45^\circ \text{ or } \frac{\pi}{4} \text{ radian}$$

Steady state

$$= \frac{\sqrt{2}}{10} \sin(t + \frac{\pi}{4})$$