ENG 381
Assignment

1. $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

when $t = 0$, $x = 0.1$ and $dx/dt = 0$

Solution

(a) using auxiliary equation method,

$m^2 + 5m + 6 = 0$,

by factorizing,

$(m+3)(m+2) = 0$

$\therefore m_1 = -3$ and $m_2 = -2$

$\therefore x = Ae^{-3t} + Be^{-2t}$

for P.I,

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

put it into the original equation,

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

equating co-efficients,

$-C + 5D + 6C = 1$ - - - (i)

$5A + 5C = 1$ ---- (i)

$-A - 5C + 6A = 0$ ---- (ii)

$-5C + 5A = 0$ ---- (iii)

$-A - 5C + 6A = 0$ ---- (ii)

$5A + 5C = 1$ ---- (i)

$5A - 5C = 0$ ---- (ii)

subtract eqn (ii) from (i)

$10C = 1$

$C = \frac{1}{10}$

from eqn (i)

$5A + 5\left(\frac{1}{10}\right) = 1$

$5A + \frac{1}{2} = 1$

$5A = \frac{1}{2}$

$A = \frac{1}{10}$

$\therefore \quad x = P.I = \frac{1}{10}[\cos t + \sin t]$

$\therefore \quad x = C.F + P.I = Ae^{-3t} + Be^{-2t} + \frac{1}{10}[\cos t + \sin$

when $t = 0$, $x = 0.1$

$$0.1 = Ae^{0} + Be^{0} + \frac{1}{10}[\cos 0 + \sin 0]$$

$$0.1 = A + B + 0.1$$
$$0 = A + B$$
$$A = -B \quad \text{-----} \quad (i)$$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} + \frac{1}{10}[-\sin t + \cos t]$$

when $t = 0$, $dx/dt = 0$,

$$0 = -3A - 2B + \frac{1}{10}[\cancel{\quad}]$$

$$0 = -3A - 2B + 0.1$$

$$-0.1 = -3A - 2B \quad \text{----} \quad (ii)$$

but from eqn (i), $A = -B$,

$$-0.1 = -3(-B) - 2B$$
$$-0.1 = +3B - 2B$$
$$-0.1 = B,$$

$\therefore \quad A = -(-0.1) = 0.1,$

$\therefore \quad x = 0.1e^{-3t} - 0.1e^{-2t} + 0.1[\cos t + \sin t],$

2. command window

```
clear
clc
close all
syms x, t
x = [0.1 * exp(-3*t)] - [0.1 * exp(-2*t)] + [0.1 * (cos(t) + sin(t))]
t = 0 : 0.01 : 15
k_n = subs (x)
plot (t, k_n)
xlabel ['time']
grid on
grid minor
axis tight
```

3. At steady state, ~~x~~ and

$$0.1\cos t + 0.1\sin t = k\sin(t + a)$$
$$0.1\cos t + 0.1\sin t = k\sin t\cos a + k\sin a\cos t$$

comparing co-efficients,

for cost, $0.1 = k\sin a$ ---- (i)

for sint, $0.1 = k\cos a$ ---- (ii)

Square ksina and kcosa and equate it to 0.1 + 0.1

$$k^2\sin^2 a + k^2\cos^2 a = 0.1 + 0.1$$
$$k^2\sin^2 a + k^2\cos^2 a = 0.2$$
$$k^2(\sin^2 a + \cos^2 a) = 0.2$$
$$k^2 = 0.2 \qquad [\sin^2 a + \cos^2 a = 1]$$
$$k = \frac{\sqrt{2}}{10},$$

for a,

$$\frac{k\sin a}{k\cos a} = \frac{0.1}{0.1}$$
$$\tan a = 1$$
$$a = \tan^{-1}[1]$$
$$a = 45^\circ \text{ or } \pi/4$$

∴ k steady state,

$$k_{ss} = \frac{\sqrt{2}}{10}\sin(\pi/4 + t)$$