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17/ENG07/1002

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1. The dynamic Modes of a body on motion performing damped force vibration is an equation (1).

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$$

given that when $t=0$, $x=0.1$ and $\frac{dx}{dt} = 0$.

a) Using the auxiliary equation Method obtain of the Mode, in form of a expression having x as a function of t.

b) With the aid of Matlab file program plot the relationship between x and t for $0 \leq t \leq 15$ time Unit Using a step size of 0.01 Unit and

c) Write the steady state solution of the Modes in form of $x = k\sin(t+a)$

Solution.

a) $$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$$

Auxiliary equation: $M^2 + 5M + 6 = 0$

$$M^2 + 2M + 3M + 6 = 0$$

$$(M+2)(M+3) = 0$$

$$M = -3 \text{ or } M = -2.$$

$$C.F = Ae^{-3x} + B^{-2x}.$$

Particular Integral

$$x = C\cos t + D\sin t$$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

Putting the value of d^2x/dt^2 & dx/dt in the equation

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

Comparing Coefficient

$[\cos t] \Rightarrow C + 5D + 6C = 1$

$[\sin t] \Rightarrow -D\cdot - 5C + 6D = 0$

$5D + 5C = 1$

$5D - 5C = 0$ --- equ

from eqn 2 $5D = 5C$.

$5C + 5C = 1$

$10C = 1$

$C = 1/10$

Substitute $C = 1/10$ into equ -.

$5D + 5\left(\frac{1}{10}\right) = 1$

$5D + 1/2 = 1$

$5D = -1/2$

$D = 1/2 \div 5$

$D = 1/2 \times 1/5 \qquad D = 1/10 = 0.1$.

General Solution $x = Ae^{-3t} + Be^{-2t} + 0.1\sin t + 0.1\cos t$

when $t = 0$; $x = 0.1$ $\quad \frac{dx}{dt} = 0$

$0.1 = A + B + 0.1$

$A + B = 0$

$\frac{dy}{dx} = -3Ae^{-3t} - 2Be^{-2t} + 0.1\sin t + 0.1\cos t$

$0 = -3A - 2B + 0.1$

from equ $B = -A$

$0 = -3A - 2(-A) - 0.1$

$0 = -A + 0.1$

$A = 0.1$.

$B = A$

$$g.s = 0.1e^{-3t} - 0.1e^{-2t} + 0.1\sin t + 0.1\cos t$$

c, $0.1\cos t + 0.1\sin t = k\sin(t+a)$ at steady flow

$$0.1\cos t + 0.1\sin t = k\sin t\cos a + k\sin a\cos t$$

Comparing Coefficient

Coefficient of $\cos t$: $0.1 = k\sin a$

Coefficient of $\sin t$: $0.1 = k\cos a$

Square $k\sin a$ and $k\cos a$ and equale it to addition.

$$k^2\sin^2 a + k^2\cos^2 a = 0.1 + 0.1$$

$$k^2(\sin^2 a + \cos^2 a) = 0.2$$

$$k^2 = 0.2 \qquad (\sin^2 a + \cos^2 a = 1)$$

$$k^2 = 2/100$$

$$k = \sqrt{2}/100$$

$$\frac{k\sin a}{k\cos a} = \frac{0.1}{0.1}$$

$$\tan a = 1$$

$$a = \tan^{-1}(1)$$

$$a = 45^\circ$$

k Steady state :

$$k_{s.s} = \frac{\sqrt{2}}{10}\sin(45 + t)$$

② Command Window

```
Clear
clc
Close all
Syms k t
k= 0.1 exp(-3*t) - 0.1* exp (-2*t) + 0.1 - cos (t) + 0.1 sin (t)
t = 0 : 0.01 : 15
k_n - subs (x)
Plot (t, x_n)
x Label ('time')
grid on
grid minor
axis tight.
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