OLOWOYO OMOSHALEWA DANIEL
17/ENG05/033

ENG 381 : ENGINEERING MATHS
MECHATRONICS

ANSWER:

QUESTION 1: Solution

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$$

In Auxiliary form

$m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$(m^2 + 2m)(3m + 6)$

$m(m+2) + 3(m+2)$

$m = -2$ & $m = -3$

$x = Ae^{-2t} + Be^{-3t}$

$P.I = \cos t$

$x = C\cos t + D\sin t$

$dx/dt = -C\sin t + D\cos t$

$d^2x/dt^2 = -(C\cos t - D\sin t)$

Substituting back in the equation

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t + 5D\cos t - 5C\sin t + 6C\cos t + 6D\sin t = \cos t$

$-C\cos t + 6C\cos t - D\sin t + 6D\sin t - 5D\cos t - 5C\sin t = \cos t$

$5C\cos t + 5D\sin t - 5C\sin t + 5D\cos t = \cos t$

$5C\cos t + 5D\cos t + 5D\sin t - 5C\sin t = \cos t$

$5C\cos t + 5D\cos t = \cos t$

$5C + 5D = 1 \quad ---(i)$

$5D\sin t - 5C = 0$

$5D - 5C = 0 \quad ---(ii)$

Using simultaneous equation.

$5C + 5D = 1$

$-5C + 5D = 0$

$10D = 1$

$D = 1/10$

$5C + 5(1/10) = 1$

$5C + 1/2 = 1$

$5C = 1 - 1/2$

$5C = 1/2$

$C = 1/10$

$G.S = Ae^{-2t} + Be^{-3t} + 1/10(\sin t + \cos t)$

When $t = 0$, $x = 0.1$ $\quad \therefore C1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}[\sin(0) + \cos(0)]$

$0.1 = A + B + 0 + \frac{1}{10}$

$A + B = 0$ $\quad$ - - (i)

when $t = 0$, $dx/dt = 0$

$dx/dt = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t)$

$0 = -2A - 3B + 0.1$

$-0.1 = -2A - 3B$

Remember $A + B = 0$ $\quad$ - - (ii)

$A = -B$ (v)

$-0.1 = -2(-B) - 3B$

$-0.1 = 2B - 3B$

$-0.1 = -B$

$B = 0.1$

Knowing that $A = -B$

$A = -0.1$

$\therefore z = -0.1e^{-2t} + 0.1e^{-3t} + \frac{1}{10}(\sin t + \cos t)$

$z = \frac{-1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}[\sin t + \cos t]$

2 Command window

```
clear
clc
close all
syms t
x = (1/15)*exp(-2*t) - (1/10)*exp(-3*t) + (1/10)(sin(t) + cos(t))
t = 0:0.01:1
xt = subs(x,t)
xtn = double(xt)
plot(t, xtn)
xlabel('t')
ylabel('x')
grid on
grid minor
grid right
```

Question 3

At steady state

$x_{ss} = x_{steady\ state} = 0.1\cos t + 0.1\sin t$

$0.1\cos t + 0.1\sin t = K\sin(t+\alpha)$

$K\sin(t+\alpha) = K\sin t\cos\alpha + K\cos t\sin\alpha$

NB coefficient of $\cos t = K\sin\alpha$

NB coefficient of $\sin t = K\cos\alpha$

When squaring both sides

$K^2\sin^2\alpha + K^2\cos^2\alpha = 0.1^2 + 0.1^2$

$K^2(\sin^2\alpha + \cos^2\alpha) = 0.02$

$K^2 = 0.02$

$K = \sqrt{0.02}$

$K = 0.144 = \sqrt{2}/10$

$\frac{K\sin\alpha}{K\cos\alpha} = \frac{0.1}{0.1} = 1$

Remember that $\sin/\cos = \tan$

$\tan\alpha = 1$

$\tan^{-1}(1) = \alpha$

$\alpha = 45^\circ$ or $\pi/4$ radian

Steady state

$= \frac{\sqrt{2}}{10}\sin(t + \pi/4)$