CHUKWUDULUE JEDIDIAH AKUOMAKACHUKWU
17/ENG03/014
ENG 381

1. $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

Assuming $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$

$\therefore \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = k^2 + 5k + 6 = 0$

$k^2 + 2k + 3k + 6 = 0$

$k(k+2) + 3(k+2) = 0$

$(k+3)(k+2) = 0$

$k+3 = 0 \qquad k+2 = 0$

$\therefore k_1 = -3$ or $k_2 = -2$

$\therefore CF \Rightarrow Ae^{-3x} + Be^{-2x}$

$P.I \Rightarrow f(x) = \cos t$

$\therefore x = C\cos t + D\sin t$

$\therefore \frac{dx}{dt} = -C\sin t + D\cos t$

$\therefore \frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$\therefore (-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) \cancel{+6(C}$
$+6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t - 5C\sin t - D\sin t + 6D\sin t = \cos t$

$\therefore 5D\cos t - C\cos t + 6C\cos t - 5C\sin t - D\sin t + 6D\sin t$

$5D + 5C = 1 \ldots\ldots (1)$

$5D - 5C = 0 \ldots\ldots (2)$

Solving simultaneously $\quad 10D = 1 \therefore D = \frac{1}{10}$

Substituting $D = \frac{1}{10}$ into (1)

$\Rightarrow 5(\frac{1}{10}) + 5C = 1 \Rightarrow \frac{1}{2} + 5C = 1$

$\Rightarrow 5C = 1 - \frac{1}{2} \qquad \Rightarrow 5C = \frac{1}{2}$

$C = \frac{1}{10}$

$\therefore P.I \Rightarrow x = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

Therefore $GS \Rightarrow P.I + C.F$

$\Rightarrow Ae^{-3x} + Be^{-2x} + \frac{1}{10}\sin t + \frac{1}{10}\cos t$

$\Rightarrow Ae^{-3x} + Be^{2x} + \frac{1}{10}(\sin t + \cos t)$

when $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

Substituting these values into the equation

$0.1 = A(1) + B(1) + \frac{1}{10}(1)$

$0.1 = A + B + 0.1$

$A + B = 0.1 - 0.1$

$A + B = 0 \quad \ldots\ldots (1)$

$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$

$0 = -3A - 2B + \frac{1}{10}$

$3A + 2B = \frac{1}{10} \quad \ldots\ldots (2)$

$A + B = 0 \quad \ldots\ldots \times 2$

$3A + 2B = 1/10 \quad \ldots\ldots \times 1$

$2A + 2B = 0$

$-(3A + 2B) = 1/10$

$\therefore -A = -0.1$

$\therefore A = 0.1$

To find B we substitute A into equ. (1)

$0.1 + B = 0$

$\therefore B = -0.1$

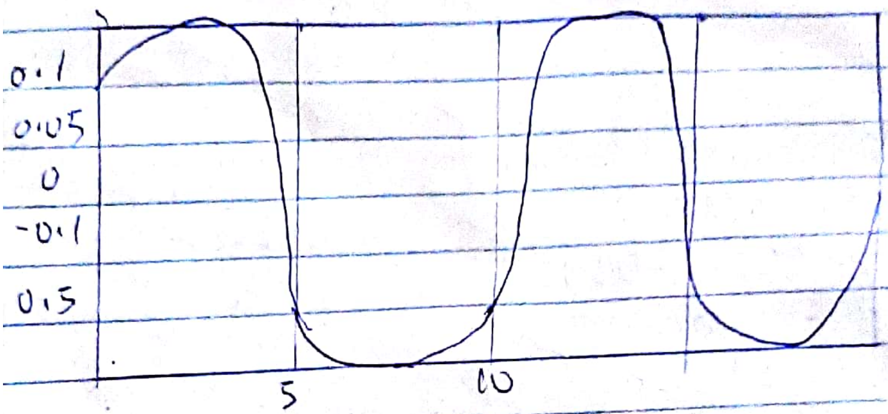
$\therefore G.S = 0.1e^{-3t} - 0.1e^{-2t} + 1/10(\sin t + \cos t)$

$= 0.1(e^{-3t} - e^{3t} + \sin t + \cos t)$

iii MATLAB

Command window

```
clear
clc
close all
Syms t
t = 0:0.01:15
x = 0.1[exp(-3*t) - exp(-2*t) + cos(t) + sin(t)]
xn = subs(x)
plot(t, xn)
axis tight
grid on
grid minor
```




$x = k\sin(t + a)$

Knowing that $x = 0.1$ at $t = 0$ and $\frac{dx}{dt} = 0$

$\frac{dx}{dt} = k\cos(t + a)$

$0 = k\cos(0 + a)$

$\therefore k\cos(a) = 0$

$0.1 = k\sin(0 + a)$

$k\sin(a) = 0.1$ . . . (1)

$\cos a = 0$

$\therefore a = \cos^{-1} = 0$

$= 90°$

Substituting a into (1)

$0.1 = k\sin(90)$

$\therefore k = \frac{0.1}{\sin(90)} = 0.1$

$\therefore x = 0.1\,[\sin(t+90)]$

Command window

```
close all
clear
clc
syms t x
  t = [0:0.01:15]
  x = 0.1*(sin(t+90))
  plot(t,x).
```

Question 3

At steady state

$+ x_{sw}$ = steady start $= 0.1\cos t + 0.1\sin t$

$0.1\cos t + 0.1\sin t = k\sin(t+a)$

$k\sin(t+a) = k\sin t\cos a + k\cos t\sin a$

NB coefficient of $\cos t = k\sin a$

NB coefficient of $\sin t = k\cos a$

- When squaring both sides

$k^2\sin^2 a + k^2\cos^3 a = 0.1^2 + 0.1^2$

$k^3(\sin^2 a + \cos^2 a) = 0.02$

$k^2 = 0.02$

$k = \sqrt{0.02}$

$k = 0.144 = \frac{\sqrt{2}}{10}$

$\frac{k\sin a}{k\cos a} = \frac{0.1}{0.1} = 1$

Remember that $\sin/\cos = \tan$

$\tan a = 1$

$\tan^{-1}(1) = a$

$a = 45°$ or $\pi/4$

Steady state

$= \frac{\sqrt{2}}{10}\sin(t + \pi/4)$