ASSIGNMENT 1

The dynamic model of a body in motion performing damped force vibration is an equation 1

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that when $t=0$, $x=0.1$ and $\frac{dx}{dt}=0$.

Using the auxillary equation method, obtain the solution of the model in form of an expression having x as the function of t.

With the aid of a MATLAB mfile program plot the relationship between x and t for $0 \leq t \leq 15$ time unit using a step size of 0.01 unit and

write the steady state solution of the model in form of

$x = k\sin(t+a)$

Solution

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Auxillary Eqn : $m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$

$(m+3)(m+2) = 0$

$m = -3$ or -2

Complementary function : $x = Ae^{-3t} + Be^{-2t}$

Particular Integral : Assume $x = C\cos t + D\sin t$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

Putting the value of d^2x/dt^2 & dx/dt in the eqn

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6((C\cos t + D\sin t) = \cos t$

comparing coefficient

Coefficient of cost : $-C + 5D + 6C = 1$ ... (1)

Coefficient of sint : $-D - 5C + 6D = 0$ ... (2)

from eqn (1) $5C + 5D = 1$ ... (3)

from eqn (2) $5D + 5C = 0$ ... (4)

From eqn (1) $5D = 1 - 5C$ ... (5)

put eqn (5) in eqn (4)

$\therefore 5D = 1 - 5C$

put the value of C in eqn(5)

$5D = 1 - 5(0.1)$

$5D = 0.5$

$D = \frac{0.5}{5} = 0.1$

Particular solution: $x = Ae^{-3t} + Be^{-2t} + 0.1\sin t + 0.1\cos t$

when $t = 0$, $x = 0.1$, $\frac{dx}{dt} = 0$

$0.1 = A + B + 0.1$

$A + B = 0$ ---------- (6)

$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} + 0.1\sin t + 0.1\cos t$

$0 = -3A - 2B + 0.1$

(from eqn (6) $B = -A$

$0 = -3A - 2(-A) + 0.1$

$0 = -A + 0.1$

$A = 0.1$

$B = -0.1$

General solution : $0.1e^{-3t} - 0.1e^{-2t} + 0.1\sin t + 0.1\cos t$

Command window

```
clear
clc
close all
syms xt
x = 0.1 * exp(-3*t) - 0.1 * exp(-2*t) + 0.1 * cos(t) + 0.1 * sin(t)
t = 0:0.01:15
kn = subs(x)
plot(t, kn)
klabel ['time']
grid on
grid minor
axis tight
```

square $k\sin\alpha$ and $k\cos\alpha$ and equate it to the addition

$$k^2\sin^2\alpha + k^2\cos^2\alpha = 0.1 + 0.1$$

$$k^2(\sin^2\alpha + \cos^2\alpha) = 0.2 \qquad [\sin^2\alpha + \cos^2\alpha = 1]$$

$$k^2 = 0.2$$

$$k^2 = \frac{2}{100}$$

$$k = \frac{\sqrt{2}}{10}$$

$$\frac{k\sin\alpha}{k\cos\alpha} = \frac{0.1}{0.1}$$

$$\tan\alpha = 1$$

$$\alpha = \tan^{-1}(1)$$

$$\alpha = 45^\circ \text{ or } \frac{\pi}{4}$$

$\therefore$ k steady state;

$$k_{ss} = \frac{\sqrt{2}}{10}\sin\left(\frac{\pi}{4} + t\right)$$