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17/ENG05/049
Mechanical Engineering ENG 381

$d^2x/dt^2 + 5\,dx/dt + 6x = \cos t$

cf $m^2 + 5m + 6 = 0$

using Completing the Square method

$m + 5m(-5/2)^2 = -6 + 25/4$

$(m + 5/2)^2 = 1/4$

Then root both side

$m + 5/2 = \pm 1/2$

$m = -3$ or -2

$A = Cf = Ae^{-3t} + Be^{-2t}$

Particular Integral

$A = C\cos t + D\sin t$

$\frac{dy}{dx} = -C\sin t + D\cos t$

$\frac{d^2y}{dx^2} = -C\cos t - D\sin t$

$(-(C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$5(C\cos t + 5D\cos t = \cos t - \cos t$ ... Eqn (1)

$-5(C\sin t + 5D\sin t = 0$

Simultaneous equation

$5C + 5D = 1$

$-5C + 5D = 0$

$10D = 1$

$D = 1/10$

P.I $= x = -1/10 \cos t + 1/10 \sin t$

$x = Ae^{-t} + Be^{-2t} + 1/10(\sin t - \cos t)$

When $t = 0$, $x = 0.1$ and $dy/dx = 0$

$0.1 = A(1) + B(1) + 1/10(-1)$

$0.1 = A + B - 0.1$

$A + B = 0.1 + 0.1$

$A + B = 0.2$ equation (3)

$dx/dt^2 = 3Ae^{-t} - 2Be^{-2t} + 1/10(\cos t + \sin t)$

$0 = 3A - 2B + 1/10(1)$

$-0.1 = 3A - 2B$

$3A + 2B = 0.1$ equation (4)

Thus

$A + B = 0.2$

$A = 0.2B$ equation (5)

Sub equation (5) into equation (4)

$3(0.2 - B) + 2B = 0.1$

$0.6 - 3B + 2B = 0.1$

$-B = -0.5$

$B = 0.5$

$A = 0.3$

Therefore

$x = 0.3e^{-3t} + 0.1e^{-2t} + 1/10(\sin t - \cos t)$

$= 0.1e^{-2t} - 0.3e^{-3t} + 1/10(\sin t - \cos t)$

Command window

```
close all;
clear all;
clc;
form x, t
t = (0:0.1:15);
x = 0.1*exp(-2*t) - 0.3*exp(-3*t). 0.1 [Sin(t) - Cos(t)]
plot (t, x)
```

ii) $x = 1c \sin(t + a)$

Knowing $x = 0.1$ at $t = 0$ and $dx/dt = 0$

$dx/dt = K\cos(t + a)$

$0 = K\cos(0 + a)$

$K = \cos(a) = 0$

$0.1 = K\sin(0 + a)$ ... equation (1)

$K \sin(a) . 0.1$

$\cos(a) = 0 \quad a = \cos^{-1}(0)$

$a = 90^\circ$

Sub a into equation (1)

$0.1 = K \sin(90)$

$K = 0.1 / \sin 90$

$K = 0.1$

$x = 0.1(\sin(t\,90))$

```
Command window
Close all
Clear all
Clc
Sign t, x
t = [0:0.1:15]
x = 0.1^t (sin(t 90)^6
Plot (t, x)
```