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$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that when $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

Using auxiliary method,

$$m^2 + 5m + 6 = 0$$

Solving for complementary function (C.F),

$$m^2 + 5m + 6 = 0$$

$$m^2 + 3m + 2m + 6$$

$$m(m+3) + 2(m+3) = 0$$

$$(m+2)(m+3) = 0$$

$$m = -2 \text{ or } m = -3$$

$$x = Ae^{mx} + Be^{mx}$$

$$x = Ae^{-2t} + Be^{-3t} \quad \text{(Complementary function)}$$

Solving for Particular Integral (P.I),

$$f(t) = \cos t$$

$$x = D\cos t + E\sin t$$

$$\frac{dx}{dt} = -D\sin t + E\cos t$$

$$\frac{d^2x}{dt^2} = -D\cos t - E\sin t$$

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$-(D\cos t + E\sin t) + 5(-D\sin t + E\cos t) + 6(D\cos t + E\sin t) = \cos t$$

$$-D\cos t - E\sin t - 5D\sin t + 5E\cos t + 6D\cos t + 6E\sin t = \cos t$$

$$(-D + 5E + 6D)\cos t + (-E - 5D - 6E\sin t) = \cos t$$

$$(5E + 5D)\cos t + (5E - 5D)\sin t = \cos t$$

Equating

$$5D + 5E = 1 \quad \cdots (i)$$

$$5E - 5D = 0 \quad \cdots (ii)$$

$5E + 5D = 1$ ---(i)

$-(5E - 5D = 0$ ---(ii)

$10D = 1$

$\therefore D = \frac{1}{10} = 0.1$

Substitute $D = \frac{1}{10}$ into eqn (ii)

$5E - 5D = 0$

$5E = 5D$

$5E = 5(\frac{1}{10})$

$5E = \frac{1}{2}$

$E = \frac{1}{2} \times \frac{1}{5}$

$E = \frac{1}{10} = 0.1$

$\therefore D = \frac{1}{10}$, $E = \frac{1}{10}$

$x = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$x = \frac{1}{10}(\cos t + \sin t)$ ——— (P.I)

$x = C.F + P.I$ ---- General solution

$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}(\cos t + \sin t)$ ---- General solution

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$

$\frac{d^2x}{dt^2} = 4Ae^{-2t} + 9Be^{-3t} + \frac{1}{10}(-\cos t - \sin t)$

When $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}(\cos(0) + \sin(0))$

$0.1 = A + B + \frac{1}{10}$

$A + B = 0.1 - 0.1$

$A + B = 0$ ---- (iii)

$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{1}{10}\sin(0) + \frac{1}{10}\cos(0)$

$0 = -2A - 3B + \frac{1}{10}$

$2A + 3B = \frac{1}{10}$ --- (iv)

Multiply eqn (iii) by 2

$A+B=0 \quad \cdots$ (iii) $\times 2$

$2A+2B=0 \quad \cdots$ (v)

$-($ $2A+3B=1/10 \quad \cdots$ (iv)

$-B = +1/10$

$B = 1/10$

Substitute $B = 1/10$ into eqn (iii)

$A+B=0$

$B = -A$ $\quad B = -A$

$A = -B$

$A = -1/10$

$$x = -\frac{1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

$$x = \frac{1}{10}(e^{-3t} - e^{-2t} + \cos t + \sin t) \quad \cdots\cdots \text{Particular solution}$$

MATLAB

m-file program

Command Window

```
close all;
clear all;
clc;
syms t
x = 1/10 * ( exp^-3*t - exp^-2*t + cos*t + sin*t)
t = 0:0.01:15
xt = subs (x, t)
xtn = double (xt)
plot (t, x, tn)
xlabel ('t')
ylabel 'x'
grid on
grid minor
```

Steady state equation

$t \geq 0 \quad x = 0.1\cos t + 0.1\sin t$

$0.1\cos t + 0.1\sin t = A\sin(t+\alpha)$

Expand the right hand side

$A\sin(t+\alpha) = A\sin t\cos\alpha + A\cos t\sin\alpha$

Coefficient of $\cos t = A\sin\alpha$

Coefficient of $\sin t = A\cos\alpha$

Squaring both sides

$A^2\sin^2\alpha + A^2\cos^2\alpha = 0.1 + 0.1^2$

$A^2(\sin^2\alpha + \cos^2\alpha) = 0.02$

$A^2 = 0.02$

$A = \sqrt{0.02} = \frac{\sqrt{2}}{\sqrt{100}}$

$A = \sqrt{2}/10$

$\frac{A\sin\alpha}{A\cos\alpha} = \frac{0.1}{0.1} = 1$

Recall $\frac{\sin\alpha}{\cos} = \tan$

$\tan\alpha = 1$

$\alpha = \tan^{-1}(1)$

$\alpha = 45^\circ / \frac{\pi}{4}$ rad

$\therefore A\sin(t+\alpha)$

$\frac{\sqrt{2}}{10}\sin(t+\frac{\pi}{4})$ - - - - Steady state equation