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Mathematics Assignment 1

a) Show that the unit of the function gives in equation [1-1] as it approaches is $\frac{a}{b}$. $f[x] = \frac{\sin ax}{bx}$.

Solution

$$\lim_{x \to 0} \frac{\sin 2x}{bx}$$

using L'Hopital's rule.

$$= \lim_{x \to 0} \frac{\sin ax}{bx} \quad \{\text{Differentiate}\}$$

$$\lim_{x \to 0} \frac{a\cos ax}{b}$$

$$\frac{a}{b} \lim_{x \to 0} \frac{\cos ax}{1}$$

$$= \frac{a}{b} \cdot \frac{\cos a[0]}{1} \quad [\text{where } \cos 0 = 1]$$

$$\frac{a}{b} \cdot \frac{1}{1}$$

$$= a/b \#$$

b) The model of a system has been developed to be as given in equation [1-2] $f(x) = 5x - 21$

Given that $\delta = 0.1$ and $\Delta\delta = 0.01$, Demonstrate in tabular form that the limit of the model as $x \to 6$ is equal to 9.

Solution

$\delta = 0.1$, $\Delta\delta = 0.01$

$6 - \delta \leq x < 6 + \delta$

$6 - \delta = 6 - 0.1 = 5.9$

$6 + \delta = 6 + 0.1 = 6.1$

$\Delta\delta = 0.01$

$\therefore 5.9 + 0.01$ and $6.1 - 0.01$

$= 5.91$ $= 6.09$ #

x	$f[x]$	x	$f[x]$
5.9	8.50	6.1	9.50
5.91	8.55	6.09	9.45
5.92	8.60	6.08	9.40
5.93	8.65	6.07	9.35
5.94	8.70	6.06	9.30
5.95	8.75	6.05	9.25
5.96	8.80	6.04	9.20
5.97	8.85	6.03	9.15
5.98	8.90	6.02	9.10
5.99	8.95	6.01	9.05
6.00	9.00	6.0	9.00

c) Show whether the function given in equation [1.3] is continuous on the interval $[-5,5]$.

$$f[x] = [25 - x^2]^{1/2}$$

Solution

$$-5 \leq x \leq 5$$

$$f(x) = [25 - x^2]^{1/2}$$

$$y = [25 - x^2]^{1/2}$$

$$y^2 = 25 - x^2$$

$$y^2 + x^2 = 25 \quad \text{[Eqn of circle]}$$

but it is a square root function

$$\lim_{x \to c} [25 - x^2]^{1/2}$$

$$= \sqrt{[25 - c^2}$$

c is a range $-5 < c < 5$

$$\Rightarrow \lim_{x \to -5^+} \sqrt{[25 - x^2}$$

let $x = a + h$

$a = -5$

from R.H.S

$$\lim_{x \to -5^+} f(x) = \lim_{h \to 0} \left[25 - [-5+h]^2\right]^{1/2}$$

$$= \left[(25 - [-5+0]^2\right]^{1/2}$$

$$= \left[25 - [25]\right]^{1/2}$$

$$= \sqrt{0} = 0$$

Taking from L.H.S

$$\lim_{x \to 5^-} f(x) = \lim_{x \to 0} \left[5 - [a-h]^2\right]^{1/2}$$

$$= \lim_{x \to 0} \left[25 - [5-h]^2\right]^{1/2}$$

$$= \left[25 - [5-0]^2\right]^{1/2}$$

$$= \sqrt{25-25} = \sqrt{0} = 0$$

Since $\lim_{x \to -5^+} f(x) = \lim_{x \to 5^-} f(x)$

it is said that the function $f(x) = [25 - x^2]^{1/2}$ is continuous at $[-5, 5]$.