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Eng 281: ENG MATHS
Computer Engineering

a. Show that the limit of the function given in Equation (1.17) as x approaches 0 is $9/8$

$$f(x) = \frac{\sin ax}{bx}$$

Soln

direct substitution

$$\lim_{x \rightarrow 0} \frac{\sin ax}{bx} = \frac{\sin a(0)}{b(0)}$$

$$= \frac{0}{0} = \text{Indeterminate}$$

using L'Hopital's rule

$$\lim_{x \rightarrow 0} \frac{\sin ax}{bx} = \frac{a \cos ax}{b}$$

$$= \frac{a \cos a(0)}{b} = \frac{a \cos 0}{b} = \frac{a}{b}$$

b. The model of a system has been developed to be given in Equation (1.2)

$$f(x) = 5x - 2$$

Given that $\delta = 0.1$ and $\Delta f = 0.01$, demonstrate, in tabular form, that the limit of the model as $x \rightarrow b$ is equal to 9.

Solution

$$\begin{aligned} \delta &= 0.1, & \Delta f &= 0.01 & 6 - 0.1 &= 5.9 \\ \xi &= 0.5, & \Delta \xi &= 0.05 & 9 - 0.5 &= 8.5 \end{aligned}$$

$0.01 + x$	$0.05 + x$	0.07	0.05
5.9	8.5	6.1	9.50
5.91	8.55	6.09	9.45
5.92	8.6	6.08	9.40
5.93	8.65	6.07	9.35
5.94	8.7	6.06	9.30
5.95	8.75	6.05	9.25
5.96	8.8	6.04	9.20
5.97	8.85	6.03	9.15
5.98	8.9	6.02	9.10
5.99	8.95	6.01	9.05
6	9	6	9

c. Show whether the function given in Equation (1.30) is continuous on the interval $[-5, 5]$.
 $f(x) = (25 - x^2)^{1/2}$

Soln

For -5 :
 $\lim_{x \rightarrow -5} (25 - x^2)^{1/2} = (25 - [-5]^2)^{1/2}$
 $= (25 - 25)^{1/2} = 0 //$

For -4 :
 $\lim_{x \rightarrow -4} (25 - x^2)^{1/2} = (25 - [-4]^2)^{1/2}$
 $= (25 - 16)^{1/2} = 3 //$

For -3 :
 $\lim_{x \rightarrow -3} (25 - x^2)^{1/2} = (25 - [-3]^2)^{1/2}$
 $= (25 - 9)^{1/2}$
 $= 4 //$

For -2 :
 $\lim_{x \rightarrow -2} (25 - x^2)^{1/2} = (25 - [-2]^2)^{1/2}$
 $= (25 - 4)^{1/2} = 4.58 //$

$$\begin{aligned} \text{For } -1: \\ \lim_{x \rightarrow -1} (25 - x^2)^{1/2} &= (25 - [-1]^2)^{1/2} \\ &= (25 - 1)^{1/2} = 4.89 \end{aligned}$$

$$\text{For } 0: \\ \lim_{x \rightarrow 0} (25 - x^2)^{1/2} = (25 - [0]^2)^{1/2} = 5$$

$$\text{For } 1: \\ \lim_{x \rightarrow 1} (25 - x^2)^{1/2} = (25 - (1)^2)^{1/2} = 4.89$$

$$\text{For } 2: \\ \lim_{x \rightarrow 2} (25 - x^2)^{1/2} = (25 - (2)^2)^{1/2} = 4.58$$

$$\text{For } 3: \\ \lim_{x \rightarrow 3} (25 - x^2)^{1/2} = (25 - (3)^2)^{1/2} = 4$$

$$\text{For } 4: \\ \lim_{x \rightarrow 4} (25 - x^2)^{1/2} = (25 - (4)^2)^{1/2} = 3$$

$$\text{For } 5: \\ \lim_{x \rightarrow 5} (25 - x^2)^{1/2} = (25 - (5)^2)^{1/2} = 0$$

∴ the function $f(x) = (25 - x^2)^{1/2}$ is continuous on the interval $[-5, 5]$.