

1 Show that the limit of the function given in equation (1.1) as x approaches 0 is a/b .

$$f(x) = \frac{\sin ax}{bx}$$

Solution

$$\lim_{x \rightarrow 0} f(x) = \frac{\sin ax}{bx}$$

$$f(x) = \frac{\sin a(0)}{b(0)} = \frac{0}{0} \text{ . undefined}$$

Using L'hopitals rule

$$\lim_{x \rightarrow 0} \frac{a \cos ax}{b}$$

$$\lim_{x \rightarrow 0} = \frac{a \cos a \cdot 0}{b} = \frac{a \cos 0}{b} = \frac{a}{b}$$

$$\therefore \lim_{x \rightarrow 0} \frac{\sin ax}{bx} = \frac{a}{b}$$

2 The model of a system has been developed to be given in equation (1.2)

$$f(x) = 5x - 21$$

Given that $\delta = \overset{0.1}{\cancel{0.01}}$ and $\Delta f = 0.01$, demonstrate in tabular form that the limit of the model as $x \rightarrow 6$ is equal to 9.

$L-\epsilon$	$a-\delta$	a	$a+\delta$	$L+\epsilon$	$L+\epsilon$ (solutions)
8.50	5.90	6.00	6.10	9.50	$f(x) = 5(x) - 21 = 5(6.10) - 21 = 9.50$
8.55	5.91	6.00	6.09	9.45	$f(x) = 5(x) - 21 = 5(6.09) - 21 = 9.45$
8.60	5.92	6.00	6.08	9.40	$f(x) = 5(x) - 21 = 5(6.08) - 21 = 9.40$
8.65	5.93	6.00	6.07	9.35	$f(x) = 5(x) - 21 = 5(6.07) - 21 = 9.35$
8.70	5.94	6.00	6.06	9.30	$f(x) = 5(x) - 21 = 5(6.06) - 21 = 9.30$
8.75	5.95	6.00	6.05	9.25	$f(x) = 5(x) - 21 = 5(6.05) - 21 = 9.25$
8.80	5.96	6.00	6.04	9.20	$f(x) = 5(x) - 21 = 5(6.04) - 21 = 9.20$
8.85	5.97	6.00	6.03	9.15	$f(x) = 5(x) - 21 = 5(6.03) - 21 = 9.15$
8.90	5.98	6.00	6.02	9.10	$f(x) = 5(x) - 21 = 5(6.02) - 21 = 9.10$
8.95	5.99	6.00	6.01	9.05	$f(x) = 5(x) - 21 = 5(6.01) - 21 = 9.05$
9.00	6.00	6.00	6.00	9.00	$f(x) = 5(x) - 21 = 5(6.00) - 21 = 9.00$

$L-\epsilon$ (Solutions)

$$f(x) = 5x - 21 = 5(5.90) - 21 = 8.50,$$

$$f(x) = 5x - 21 = 5(5.91) - 21 = 8.55,$$

$$f(x) = 5x - 21 = 5(5.92) - 21 = 8.60,$$

$$f(x) = 5x - 21 = 5(5.93) - 21 = 8.65,$$

$$f(x) = 5x - 21 = 5(5.94) - 21 = 8.70,$$

$$f(x) = 5x - 21 = 5(5.95) - 21 = 8.75,$$

$$f(x) = 5x - 21 = 5(5.96) - 21 = 8.80,$$

$L+\epsilon$ (solutions)

$$f(x) = 5(x) - 21 = 5(6.10) - 21 = 9.50$$

$$f(x) = 5(x) - 21 = 5(6.09) - 21 = 9.45$$

$$f(x) = 5(x) - 21 = 5(6.08) - 21 = 9.40$$

$$f(x) = 5(x) - 21 = 5(6.07) - 21 = 9.35$$

$$f(x) = 5(x) - 21 = 5(6.06) - 21 = 9.30$$

$$f(x) = 5(x) - 21 = 5(6.05) - 21 = 9.25$$

$$f(x) = 5(x) - 21 = 5(6.04) - 21 = 9.20$$

$$f(x) = 5(x) - 21 = 5(6.03) - 21 = 9.15$$

$$f(x) = 5(x) - 21 = 5(6.02) - 21 = 9.10$$

$$f(x) = 5(x) - 21 = 5(6.01) - 21 = 9.05$$

$$f(x) = 5(x) - 21 = 5(6.00) - 21 = 9.00$$

3 Show whether the function in equation (1.3) is continuous on the interval $(-5, 5)$ $f(x) = (25-x^2)^{1/2}$

Solution

$$f(x) = (25-x^2)^{1/2} \quad [-5, 5] \Rightarrow f(x) = \sqrt{25-x^2}$$

$$f(x) = \sqrt{25-(5)^2} = 0, \quad f(x) = f(4) = \sqrt{25-(4)^2} = 9$$

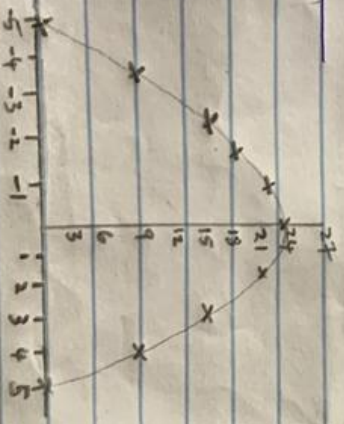
$$f(x) = \sqrt{25-(3)^2} = 16, \quad f(x) = \sqrt{25-(2)^2} = 21$$

$$f(x) = \sqrt{25-(1)^2} = 24, \quad f(x) = \sqrt{25-(0)^2} = 25$$

$$f(x) = \sqrt{25-(0)^2} = 25, \quad f(x) = \sqrt{25-(1)^2} = 24$$

$$f(x) = \sqrt{25-(3)^2} = 16, \quad f(x) = \sqrt{25-(4)^2} = 9$$

$$f(x) = \sqrt{25-(5)^2} = 0$$



f is continuous at the interval of $(-5, 5)$