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CIVIL ENGINEERING
ENG 281 ASSIGNMENT

1. Show that the limit of the function given in equation (1.1) as x approaches 0 is a/b

$$f(x) = \frac{\sin ax}{bx} \quad ---- (1.1)$$

Soln

$$\lim_{x \to 0} \frac{\sin ax}{bx}$$

Since $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$$\lim_{x \to 0} \frac{\sin ax}{ax} \times \frac{1}{b} \times a = \lim_{x \to 0} \frac{\sin ax}{bx}$$

Since $\lim_{x \to 0} \frac{\sin x}{x} = 1$ then $\frac{\sin ax}{ax} = 1$

Therefore,

$$1 \times \frac{1}{b} \times a = \frac{a}{b}$$

$$\therefore \lim_{x \to 0} \frac{\sin ax}{bx} = \frac{a}{b}$$

2. The model of a system has been developed to be as given $f(x) = 5x - 21$, Given that $\delta = 0.1$ and $\Delta\delta = 0.01$. Demonstrate in tabular form, that the limit of the model as $x \to 6$ equal to 9

Soln

$\sigma = 0.1$ and $\Delta\sigma = 0.01$

$6 - \sigma < x < 6 + \sigma$

$\therefore 6 - \sigma = 6 - 0.1 = 5.9$

$6 + \sigma = 6 + 0.1 = 6.1$

$\Delta\sigma = 0.01 \quad \therefore 5.9 + 0.01$

$= 5.91$

&

$6.1 - 0.01$

$= 6.09$

x	$f(x)$	x	$f(x)$
5.9	8.50	6.1	9.50
5.91	8.55	6.09	9.45
5.92	8.60	6.08	9.40
5.93	8.65	6.07	9.35
5.94	8.70	6.06	9.30
5.95	8.75	6.05	9.25
5.96	8.80	6.04	9.20
5.97	8.85	6.03	9.15
5.98	8.90	6.02	9.10
5.99	8.95	6.01	9.05
6.00	9.00	6.00	9.00

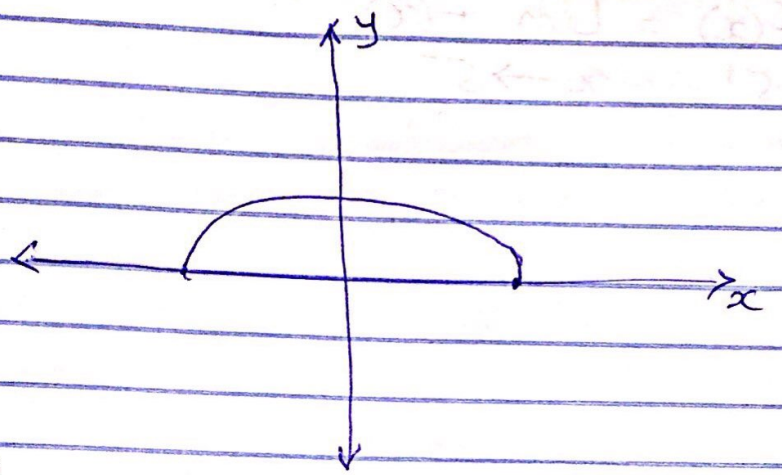
3. Show whether the function in equation (1.3) is continuous on the interval $(-5, 5)$

Soln $\quad f(x) = (25 - x^2)^{1/2} \quad ---(1.3)$

$-5 \leq x \leq 5$

$f(x) = (25 - x^2)^{1/2}$

$$y = (25 - x^2)^{1/2}$$
$$y^2 = 25 - x^2$$
$$y^2 + x^2 = 25$$



Since it is a square root function,

$$\lim_{x \to c} (25 - x^2)^{1/2} = \sqrt{25 - c^2}$$

C is in range of $-5 \leq c \leq 5$

$$\lim_{x \to -5^+} \sqrt{25 - x^2}$$

Let $x = a + h$

$a = -5$

Taking at R.H.S

$$\lim_{x \to -5^+} f(x) = \lim_{h \to 0} [25 - (-5 + h)^2]^{1/2}$$
$$= [25 - (-5 + 0)^2]^{1/2}$$
$$= [25 - (25)]^{1/2}$$
$$= \sqrt{0} = 0$$

Taking at L.H.S

$$\lim_{x \to 5^-} f(x) = \lim_{h \to 0} [25 - (a - h)^2]^{1/2}$$
$$= \lim_{h \to 0} [25 - (5 - h)^2]^{1/2}$$
$$= [25 - (5 - 0)^2]^{1/2}$$

$= \sqrt{25 - 25}$

$= \sqrt{0} = 0$

Since $\lim_{x \to -5^+} f(x) = \lim_{x \to 5^-} f(x)$

Therefore the function $f(x) = (25 - x^2)^{1/2}$ is continuous at $(-5, 5)$.