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The dynamic motion of a body in motion performing damped force vibration is an equation (1)

$$\frac{d^2x}{dt^2} + \frac{3dx}{dt} + 6x = \cos t.$$

given that when $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$.

a) Using the auxiliary equation method obtain an the model in form of a expression having x as a function of t.

(b) With the aid of matlab file program plot the relationship between x and t for [illegible] time unit using a step size of 0.01 unit and

(c) Write the steady state solution of the model in form of $x = k\cos(t + a)$

Solution

a) $\frac{d^2x}{dt^2} + \frac{3dx}{dt} + 6x = \cos t$

Auxiliary Equation $m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$

$(m+2)(m+3) = 0$

$m = -2$ or $m = -3$

$CF = Ae^{-3x} + Be^{-2x}$

Particular Integral Assume the Right Hand Side =

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

Putting the value of $\frac{d^2x}{dt^2}$ & $\frac{dx}{dt}$ in the equation

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t)$

Comparing the coefficient

$(\cos t) = C + 5D + 6C = 1$

$(\sin t) = -D - 5C + 6D = 0$

$5D + 5C = 1$

$5D - 5C = 0$ --- equation

From Equation 2 $5D = 5C$

$5C + 5C = 1$

$10C = 1$

$C = 1/10$

Substitute $C = 1/10$ into equation 1

$5D + \frac{5(1)}{10} = 1$

$5D + 1/2 = 1$

$5D = -1/2$

$D = 1/2 \div 5$

$D = 1/2 \times 1/5$, $D = 1/10 = 0.1$

P.g. $x = Ae^{-3t} + Be^{-2t} + 0.1\sin t + 0.1\cos t$

when $t = 0$; $x = 0.1$ $\frac{dx}{dt} = 0$

$0.1 = A + B + 0.1$

$A + B = 0$

$\frac{dy}{dx} = -3Ae^{-3t} - 2Be^{-4} + 0.1\sin t + 0.1\cos t$

$0 = -3A - 2B + 0$

From Equation (6) $\rightarrow B = -A$

$0 = -3A - 3B + 0$

$0 = 3A - 2(-A) + 0.1$

$0 = -A + 0.1$

$A = 0.1$

$B = A$

$y.s = 0.1e^{-3t} - 0.1e^{-4t} + 0.1\sin t + 0.1\cos t$

c) $0.1\cos t + 0.1\sin t = k\sin(t + a)$ at steady state

$0.1\cos t + 0.1\sin t = k\sin t\cos a + k\sin a\cos t$

Comparing coefficient

Coefficient of $\cos t$ $1 = 0.1 = k\sin a$

Coefficient of $\sin t$ $= 0.1 = k\cos a$

Square $k\sin\alpha$ and $k\cos\alpha$ and equate it to the addition.

$$k^2\sin^2\alpha + k^2\cos^2\alpha = 0.1 + 0.1$$

$$k^2(\sin^2\alpha + \cos^2\alpha) = 0.2$$

$$k^2 = 0.2 \qquad (\sin^2\alpha + \cos^2\alpha = 1)$$

$$k^2 = \frac{2}{10}$$

$$k = \sqrt{\frac{2}{100}}$$

$$\frac{k\sin\alpha}{k\cos\alpha} = \frac{0.1}{0.1}$$

$$\tan\alpha = 1$$

$$\alpha = \tan^{-1}(1)$$

$$\alpha = 45°$$

K steady state

$$V_{ss} = \frac{\sqrt{2}}{10}\sin(45t)$$