

UNADGU CHINAGOROM - M.  
W/ENG 0610P6.

MECHANICAL ENGR.

ENC 381

$$\textcircled{1} \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$m^2 + 5m + 6 = 0$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$m+2 = 0 \quad m+3 = 0$$

$$m = -2 \quad \text{or} \quad m = -3$$

$$\therefore \text{C.F.} = Ae^{-2t} + Be^{-3t}$$

$$\text{P.I.} = \cos t$$

$$x = (\cos t + D \sin t)$$

$$\frac{dx}{dt} = -(\sin t + D \cos t)$$

$$\frac{d^2x}{dt^2} = -(\cos t - D \sin t)$$

$$= -(\cos t - D \sin t) + 5(-\sin t + D \cos t) + 6(\cos t + D \sin t) = \cos t$$

$$= -\cos t + D \sin t - 5 \sin t + 5D \cos t + 6 \cos t + 6D \sin t = \cos t$$

$$(-\cos t + 5D \cos t + 6 \cos t) + (-D \sin t + 5 \sin t + 6D \sin t)$$

$$= \cos t$$

$$5 \cos t + 5D \cos t = \cos t$$

$$5D \sin t - 5 \sin t = 0$$

$$5C + 5D = 1$$

$$-5C + 5D = 0$$

$$\cos = 1$$

$$D = 1/10$$

$$5C + 5D = 1$$

$$5C + 5/10 = 1$$

$$C = 1/10$$

$$\therefore \text{P.I.} = 1/10 (\cos t + \sin t)$$

$$\therefore x = \text{C.F.} + \text{P.I.}$$

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} (\cos t + \sin t)$$

when  $t=0$   $x=0.1$  &  $\frac{dx}{dt}=0$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10} (\cos 0 + \sin 0)$$

$$0.1 = A + B + 0.1$$

$$A + B = 0.1 + 0.1$$

$$A + B = 0.2 \text{ --- eqn.}$$

$$\frac{dx}{dt} = -(\sin t + \cos t)$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + \frac{1}{10} (\cos t + \sin t)$$

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} + \frac{1}{10} (\cos 0 + \sin 0)$$

$$= -2A + 3B + 0.1$$

$$= 3A + 2B = 0.1 \text{ --- (2)}$$

$$A + B = 0.2$$

$$A = 0.2 - B \text{ --- (3)}$$

Sub eqn (3) into eqn (2)

$$3(0.2 - B) + 2B = 0.1$$

$$0.6 - 3B + 2B = 0.1$$

$$-B = -0.5$$

$$B = 0.5$$

$$A = 0.3$$

$$x = -0.3e^{-2t} + 0.5e^{-3t} + \frac{1}{10} (\cos t + \sin t)$$

Command Window

clear

clc

close all

syms x; t;

$$t = [0:0.1:15];$$

$$x = 0.1 (\sin(t + 90))$$

$$\text{plot}(t, x)$$

$$c. \quad x = k \sin(t + \phi)$$

$$x = 0.1 \quad t = 0$$

$$\frac{dx}{dt} = k \cos(t + \phi)$$

$$0 = k \cos(0 + \phi)$$

$$0 = k \cos \phi$$

$$0.1 = k \sin(\phi)$$

$$k \sin \phi = 0.1$$

$$\cos \phi = 0$$

$$\phi = 90^\circ$$

$$\phi = \cos^{-1}(0)$$

$$\phi = 90^\circ$$

Sub into eqn (1)

$$0.1 = k \sin 90^\circ$$

$$k = 0.1 / \sin 90^\circ$$

$$k = 0.1$$

$$x = 0.1 \sin(t + 90^\circ)$$