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Civil Engineering
17/ENG03/017
ENG 381

Soln

(a) $\frac{d^2x}{dt} + 5\frac{dx}{dt} + 6x = \cos t$ — x

taking the complimentary function by auxilliary method

$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$ — xx

$x_c = Ae^{kt}$ — (i)

$\dot{x}_c = Ake^{kt}$ — (ii)

$\ddot{x}_c = Ak^2e^{kt}$ — (iii)

Sub eq (i), (ii) and (iii) into xx

$Ak^2e^{kt} + 5(Ake^{kt}) + 6(Ae^{kt}) = 0$

$\frac{Ae^{kt}}{Ae^{kt}}(k^2 + 5k + 6) = \frac{0}{Ae^{kt}}$

$k^2 + 5k + 6$

$k^2 + 3k + 2k + 6 = 0$

$k(k+3) + 2(k+3) = 0$

$(k+3)(k+2)$

$k = -2, \quad k = -3$

$C.F = Ae^{-2t} + Be^{-3t}$

$f(x) = \cos t$

Partial integral $x = C\cos t + D\sin t$

$\dot{x} = -C\sin t + D\cos t$

$\ddot{x} = -C\cos t - D\sin t$

Sub into eq x

$(-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t + 0\sin t$

$\therefore -C\cos t + 5D\cos t + 6C\cos t = \cos t$

$-C - D\sin t - 5C\sin t + 6D\sin t = 0\sin t$

$-D - 5C + 6D = 0$

$5D = 5C$

$D = C$

$-C\cos t + 5D\cos t + 6C\cos t = \cos t$

$-C + 5D + 6C = 1$

$5D + 5C = 1$

$5D + 5D = 1$

$10D = 1$

$D = \frac{1}{10}$

$C = \frac{1}{10}$

$G.S = x_{cf} + x_p$

$G.S, \; x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}[\cos t + \sin t]$

Given $t = 0, \; x = 0.1, \; \frac{dx}{dt} = 0$

When $t = 0$

$x = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}[\cos(0) + \sin(0)]$

$x = 0$

$x = 0.1$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}(\cos(0) + \sin(0))$

$0.1 = \frac{1}{10} + A + B$

$A + B = 0 \quad --(i)$

When $t = 0, \; \dot{x} = 0$

$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} + \frac{1}{10}(-\sin(0) + \cos(0))$

$0 = -2A - 3B + 0.1 \quad --(ii)$

$A + B = 0 \quad --(i)$

$2A + 3B = 0.1 \quad --(ii)$

From eq i, $A = -B$

Sub in eq ii

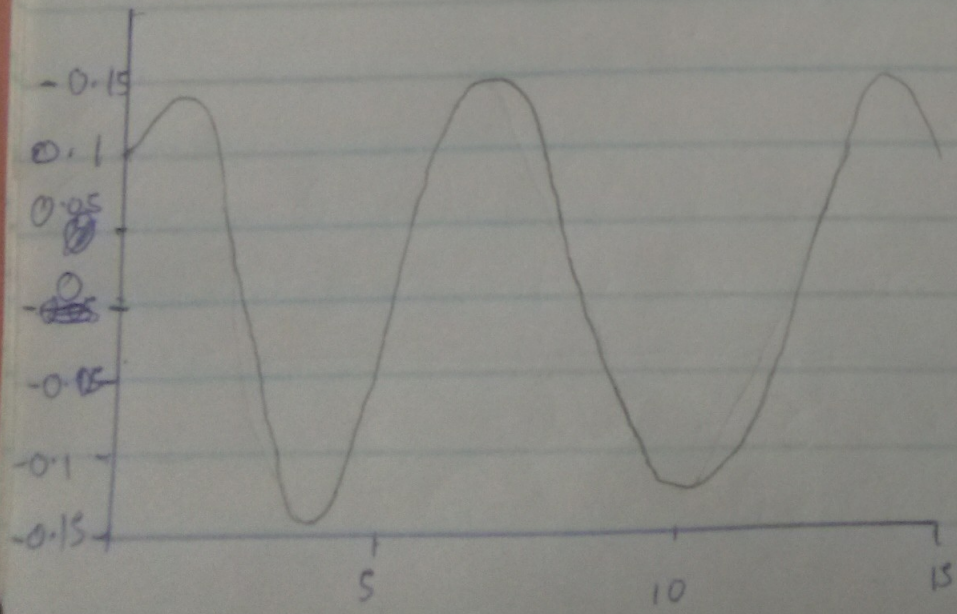
$2(-B) + 3B = 0.1$

$B = 0.1$

$A = -0.1$

$$x = -0.1e^{-2t} + 0.1e^{-3t} + \frac{1}{10}(\cos t + \sin t)$$

- Commandwindow
- clear
- clc
- close all
- syms t
- t = (0 : 0.01 : 15)
- x = -0.1*exp((-2*t)) + 0.1*exp((-3*t)) + (1/10)*(cos(t) + sin(t))
- xn = Subs (x)
- plot (t, xn)



c) $x_{ss} = x_{t\to\infty} = 0.1\cos t + 0.1\sin t = k\sin(t+\alpha)$

$= k\sin t\cos t\alpha + k\cos t\sin\alpha$

$0.1 = k\sin\alpha \; ; \; 0.1 = k\cos\alpha$

$k^2\sin^2\alpha + k^2\cos^2\alpha = \frac{1}{100} + \frac{1}{100}$

$k^2(\sin^2\alpha + \cos^2\alpha) = \frac{2}{100}$

$k^2 = \frac{2}{200}$ (since $\sin^2\alpha + \cos^2\alpha = 1$)

$k = \frac{\sqrt{2}}{10}$

$\frac{k\sin\alpha}{k\cos\alpha} = \frac{0.1}{0.1} = 1$

then $\alpha = 1, \; \alpha = \pi/4$

$x_{ss} = \frac{\sqrt{2}}{10}\sin(t + \pi/4)$

The steady state solution of the model is $\frac{\sqrt{2}}{10}\sin(t+\pi/4)$