

ENG 381

Assignment

$$1. \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

when $t=0$, $x=0.1$ and $\frac{dx}{dt} = 0$

Solution

(a) using auxiliary equation method,

$$m^2 + 5m + 6 = 0,$$

by factorizing,

$$(m+3)(m+2) = 0$$

$$\therefore m_1 = -3 \quad \text{and} \quad m_2 = -2$$

$$\therefore x = Ae^{-3t} + Be^{-2t}$$

for P.I,

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

put it into the original equation,

$$-C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

equating co-efficients,

$$-C + 5D + 6C = 1 \quad \dots \dots (i)$$

$$-D - 5C + 6D = 0 \quad \text{--- (ii)}$$

$$5D + 5C = 1 \quad \text{--- (i)}$$

$$5D - 5C = 0 \quad \text{--- (ii)}$$

subtract eqn (ii) from (i)

$$10C = 1$$

$$C = \frac{1}{10}$$

from eqn (i)

$$5D + 5\left(\frac{1}{10}\right) = 1$$

$$5D + \frac{1}{2} = 1$$

$$5D = \frac{1}{2}$$

$$D = \frac{1}{10}$$

$$\therefore x = P.I = \frac{1}{10} [\cos t + \sin t]$$

$$\therefore x = C.F + P.I = Ae^{-3t} + Be^{-2t} + \frac{1}{10} [\cos t + \sin t]$$

when $t = 0, x = 0.1$

$$0.1 = Ae^0 + Be^0 + \frac{1}{10} [\cos 0 + \sin 0]$$

$$0.1 = A + B + 0.1$$

$$0 = A + B$$

$$A = -B \quad \text{--- (ii)}$$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} + \frac{1}{10} [-\sin t + \cos t]$$

when $t = 0, dx/dt = 0,$

$$0 = -3A - 2B + \frac{1}{10} [1]$$

$$0 = -3A - 2B + 0.1$$

$$-0.1 = -3A - 2B \quad \text{--- (iv)}$$

but from eqn (ii), $A = -B,$

$$-0.1 = -3(-B) - 2B$$

$$-0.1 = +3B - 2B$$

$$-0.1 = B,$$

$$\therefore A = -(-0.1) = 0.1,$$

sin t]

$$\therefore x = 0.1e^{-3t} - 0.1e^{-2t} + 0.1 [\cos t + \sin t],$$