Bello Folashade Shakirat
17/mmsol/083
Civil Engineering
Eng 381

The dynamic model of a body in motion performing damped forces vibration is as below

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t \qquad \text{- Equation 1}$$

Given that $t = 0$, $x = 0.1$ and $dx/dt = 0$

a) Using the Auxiliary Equation method, obtain the solution of the model in form of an expression having x as a function of t

b) With the aid of a matlab m-file program, plot the relationship between x & t for $0 \le t \le 15$ time unit using a step size of 0.01 unit

c) Write the steady state solution of the model in form of $x = R\sin(t - \alpha)$

Soln

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

In auxiliary form

$m^2 + 5m + 6 = 0$

$(m+2)(m+3) = 0$

$m+2 = 0$ or $m+3 = 0$

$m = -2$ or -3

$x = Ae^{-2t} + Be^{-3t}$

$P.I = \cos t$

$x = C\cos t + D\sin t$

$dx/dt = -C\sin t + D\cos t$

$\frac{dx^2}{dt^2} = -C\cos t - D\sin t$

Sub into the equation

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6[C\cos t + D\sin t] = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$-C\cos t + 6C\cos t - D\sin t + 6D\sin t - 5C\sin t + 5D\cos t = \cos t$

$5C\cos t + 5D\sin t - 5C\sin t + 5D\cos t = \cos t$

$5C\cos t + 5D\cos t + 5D\sin t - 5C\sin t = \cos t$

$5C\cos t + 5D\cos t = \cos t$

$5C + 5D = 1 \quad ---(i)$

$5D\sin t - 5C\sin t = \cos t$

$5D - 5C = 0$

Using simultaneous equations,

$5C + 5D = 1 \quad ---(i)$

$-5C + 5D = 0 \quad ---(ii)$

$10D = 1$

$D = 1/10$

$5C + 5(1/10)$

$5C + 1/2 = 1$

$5C = 1 - 1/2$

$5C = 1/2$

$C = 1/10$

General solution.

$Ae^{-2t} + Be^{-3t} + 1/10[\sin t + \cos t]$

When $t = 0$, $x = 0.1$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 1/10[\sin(0) + \cos(0)]$

$0.1 = A + B + 0 + 1/10$

$A + B = 0 \quad --(iii)$

When $t = 0$ $dx/dt = 0$

$dx/dt = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t)$

$0 = -2A - 3B + 0.1$

$-0.1 = -2A - 3B \quad ---(iv)$

Recall $A + B = 0 \quad ---(iii)$

$A = -B \quad ---(v)$

$-0.1 = -2(-B) - 3B$

$-0.1 = 2B - 3B$

$-0.1 = -B$

$B = 0.1$

Recall that $A = -B$

$A = -0.1$

$\therefore x = -0.1e^{-2t} + 0.1e^{-3t} + 1/10[\sin t + \cos t]$

2 Command window

```
clear
clc
close all
syms t
x=(1/10*exp(-2*t))-(1/10*exp(-3*t))+(1/10*[sin(t)+cos(t)])
t=0:0.01:15
xt = subs(x,t)
xtn = double(xt)
plot(t,xtn)
x label('t')
y label('x')
grid on
grid minor
grid right
```

At steady state

$$x = x_{\text{steady state}} = 0.1\cos t + 0.1\sin t \quad (t \to \infty)$$

$$0.1\cos t + 0.1\sin t = K\sin(t+a)$$

$$K\sin(t+a) = K\sin t\cos a + K\cos t\sin a$$

Co-efficient of $\cos t = K\sin a$

Co-efficient of $\sin t = K\cos a$

Square both side

$$K^2\sin^2 a + K^2\cos^2 a = 0.1^2 + 0.1^2$$

$$K^2(\sin^2 a + \cos^2 a) = 0.02$$

$$K^2 = 0.02$$

$$K = \sqrt{0.02}$$

$$K = 0.1414 = \sqrt{2}/10$$

$$\frac{K\sin a}{K\cos a} = \frac{0.1}{0.1} = 1$$

Recall that $\sin/\cos = \tan$

$\tan\theta = 1$

$\tan^{-1}(1) = \theta$

$\theta = 45^\circ$ or $\pi/4$ radians

Steady state

$= \frac{\sqrt{2}}{10} \sin(t + \pi/4)$