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17/ENG04/038
Elect | Elect

ENG 381

Solution to Assignment 1

$CF = m^2 + 5m + 6 = 0$

$(m+3)(m+2) = 0$

$m_1 = -3 \; ; \; m_2 = -2$

$x = Ae^{-3t} + Be^{-2t}$

$f(t) = \cos t$

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\cos t$

Substituting

$= -C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$= \cos t(-C + 5D + 6C) + \sin t(-D + 6D - 5C) = \cos t$

$\cos t(5D + 5C) + \sin t(5D - 5C) = \cos t$

$5D + 5C = 1$

$5D - 5C = 0$

$10D = 1$

$D = \frac{1}{10}$

to find C

$5\left(\frac{1}{10}\right) + 5C = 1$

$\frac{1}{2} + 5C = 1$

$C = \frac{1}{10}$

G.S $= x = Ae^{-3t} + Be^{-2t} + 0.1\cos t + 0.1\sin t$

Subst $(x = 0.1), t = 0$

$0.1 = Ae^{-0} + Be^{-0} + 0.1\cos(0) + 0.1\sin(0)$

$0.1 = A + B + 0.1$

$A + B = 0$ —— eqtn (1)

$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - 0.1\sin t + 0.1\cos t$

$0 = -3A - 2B + 0.1$

$3A + 2B = 0.1$ —— eqtn (2)

$A + B = 0 \quad \text{— x2}$

$3A + 2B = 0.1 \quad \text{— 1}$

$2A + 2B = 0$

$3A + 2B = 0.1$

$-A = -0.1$

$A = 0.1$

to find B

$0.1 + B = 0$

$B = -0.1$

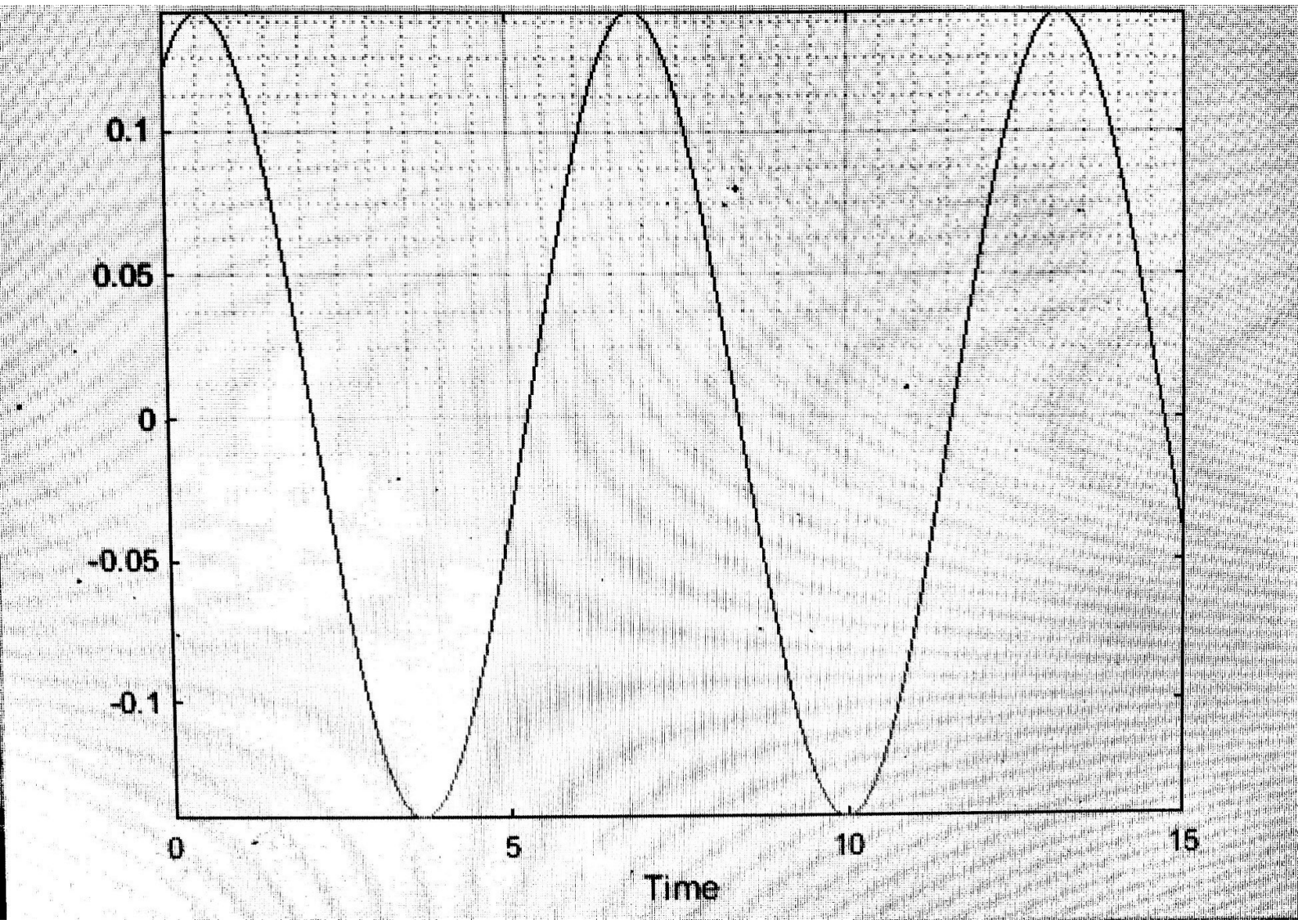
$$G.S = 0.1e^{-3t} - 0.1e^{-2t} + 0.1\cos t + 0.1\sin t$$

$$= 0.1\left(e^{-3t} - e^{-2t} + \cos t + \sin t\right)$$

ii. MATLAB File

Command window

```
clear
clc
close all
t = 0:0.01:15
x = 0.1 * [exp(-3*t) - exp(-2*t) + cos(t) + sin(t)]
xn = subs(x)
Plot(t, xn)
axis tight
grid on
grid minor
```



III $x = x_h \Rightarrow \infty = 0.1\cos t + 0.1\sin t = K\sin(t+\alpha)$

$x_{ss} = K\sin t\cos\alpha + K\cos t\sin\alpha$

$0.1 = K\sin\alpha \;;\; 0.1 = K\cos\alpha$

$0+0: \; K^2\sin^2\alpha + K^2\cos^2\alpha = \frac{1}{100} + \frac{1}{100}$

$K^2(\sin^2\alpha + \cos^2\alpha) = \frac{2}{100}$

$K^2 = \frac{2}{100}$ (since $\sin^2\alpha + \cos^2\alpha = 1$)

$K = \frac{\sqrt{2}}{100}$

$\frac{K\sin\alpha}{K\cos\alpha} - \frac{0.1}{0.1} = 1$

$\tan\alpha = 1, \; \alpha = \pi/4$

$x_{ss} = \frac{\sqrt{2}}{10}\tan\left(t + \pi/4\right)$

The steady-state solution of the model

$\frac{\sqrt{2}}{10}\sin\left(t + \frac{\pi}{4}\right)$.