

Onaechie Tochukwu Francis  
17/ENG.06/067

Mechanical Engineering

$$d^2x/dt^2 + 5 dx/dt + 6x = \cos t$$

$$m^2 + 5m + 6 = 0$$

$$(m^2 + 2m)(3m + 6) \quad m(m+2) + 3(m+2)$$

$$m = -2, m = -3$$

$$x = Ae^{-2t} + Be^{-3t}$$

$$PI = \cos t$$

$$x = C \cos t + D \sin t, \quad dx/dt = -C \sin t + D \cos t, \quad d^2x/dt^2 = -C \cos t - D \sin t$$

$$-C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$-C \cos t + 6C \cos t - D \sin t + 6D \sin t - 5C \sin t + 5D \cos t = \cos t$$

$$5C \cos t + 5D \cos t = \cos t$$

$$5C + 5D = 1 \quad (1)$$

$$-5C \sin t + 5D \sin t = 0$$

$$-5C + 5D = 0 \quad (2)$$

$$5C + 5D = 1$$

$$-5C + 5D = 0$$

$$5C + 5(1/5) = 1$$

$$C = 1/2 \times 1/5$$

$$= 1/10$$

$$\Rightarrow D = 1$$

$$D = 1/10$$

$$PI = 1/10 \cos t + 1/10 \sin t$$

$$y = Ae^{-2t} + Be^{-3t} + 1/10 [\sin t + \cos t]$$

$$\text{when } t=0, x=0.1$$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 1/10 [\sin(0) + \cos(0)]$$

$$0.1 = A + B + 1/10$$

$$A + B = 0 \quad (1)$$

$$\text{when } t=0, dx/dt = 0$$

$$dx/dt = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t)$$

$$0 = 2A - 3B + 0.1$$

$$-0.1 = 2A - 3B, \quad \text{Recall } A + B = 0$$

$$A = -B$$

$$-0.1 = 2(-B) - 3B$$

$$-0.1 = 2B - 3B$$

$$-0.1 = -B, \quad B = 0.1$$

Since  $A = -B$

$$A = 0.1$$

$$x = -0.1e^{-2t} + 0.1e^{-3t} + \frac{1}{10}[\sin t + \cos t]$$

$$1.) \quad x = \frac{-1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}(\sin t + \cos t)$$

(2) Command window

clear

clc

close all

syms t

$$6.) \quad x = (1/10 * \exp(-2*t)) - 1/10 * \exp(-3*t) + 1/10 * (\sin(t) + \cos(t))$$

$$t = 0:0.01:15$$

x\_t = subs(x,t)

x\_tn = double(x\_t)

Plot(t, x\_tn)

x\_label('t')

y\_label('x')

grid on

grid minor

grid right

3.) At steady state

$$t \rightarrow \infty = \text{steady state} = 0.1 \cos t + 0.1 \sin t$$

$$0.1 \cos t + 0.1 \sin t = K \sin(t + \alpha)$$

$$K \sin(t + \alpha) = K \sin t \cos \alpha + K \cos t \sin \alpha$$

$$\text{Coefficient of } \cos t = K \sin \alpha$$

$$\text{Coefficient of } \sin t = K \cos \alpha$$

$$K^2 \sin^2 \alpha + K^2 \cos^2 \alpha = 0.1^2 + 0.1^2$$

$$K^2 (\sin^2 \alpha + \cos^2 \alpha) = 0.02$$

$$K^2 = 0.02 \quad K = \sqrt{0.02}$$

$$K = 0.141 = \frac{\sqrt{2}}{10}$$

$$\frac{K \sin a}{K \cos a} = \frac{0.1}{0.1} = 1$$

$$K \cos a = 0.1$$

remember;  $\sin/\cos = \tan$

$$\tan a = 1$$

$$\tan^{-1}(1) = a$$

$$a = 45^\circ \text{ or } \pi/4 \text{ radians}$$

$$\text{steady state} = \frac{\sqrt{2}}{10} \sin(t + \pi/4)$$

