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MECHANICAL ENGINEERING
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$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$

① in auxiliary form

C.F: $m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2)$

$m = -2$ or $m = -3$

$x = Ae^{-2t} + Be^{-3t}$

P.I. $\cos t$

$x = C\cos t + D\sin t$

$dx/dt = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$\therefore -C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$-C\cos t + 6C\cos t - D\sin t + 6D\sin t - 5C\sin t + 5D\cos t = \cos t$

$5C\cos t + 5D\sin t + 5D\cos t - 5C\sin t = \cos t$

$5C\cos t + 5D\cos t = \cos t$

$5C + 5D = 1 \quad \text{---} ①$

$5D - 5C = 0 \quad \text{---} ②$

using simultaneous equation

$5C + 5D = 1$

$-5C + 5D = 0$

$10D = 1$

$D = 1/10$

Substituting D into eqn ①.

$5C + 5(1/10) = 1$

$5C = 1/2$

$C = 1/10$

$x = Ae^{-2t} + Be^{-3t} + 1/10(\sin t + \cos t)$

when $t = 0$, $x = 0.1$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 1/10(\sin 0 + \cos 0)$

$0.1 = A + B + 0 + 1/10$

A+B= 0 ---③

when t=0, dx/dt = 0.

$dx/dt = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t)$

$0 = -2A - 3B + 0.1$

$-0.1 = -2A - 3B$

recall, A+B=0

$A = -B$ ---④

$-0.1 = -2(-B) - 3B$

$-0.1 = 2B - 3B$

$-0.1 = -B$

$B = 0.1$

from eqn (4).

$A = -0.1$

$x = -0.1e^{-2t} + 0.1e^{-3t} + 1/10(\sin t + \cos t)$

2. Command window

```
clear
clc
close all
syms t
x= (1/10*exp(-2*t)-(1/10*exp(-3*t)+(1/10(sin t)+1cos t))
t = 0:0.01:15
xt = subs(x,t)
xtn = double(xt)
plot(t,x,tn)
xlabel('t')
ylabel('x')
grid on
grid minor
grid right
```

3. At steady state.

$t \to \infty$ - steady state $= 0.1\cos t + 0.1\sin t$.

$0.1\cos t + 0.1\sin t = k\sin(t+a)$

$B\sin(t+a) = k\sin t\cos a + k\cos t\sin a$

coefficient of cost = $k\sin a$.

coefficient of sint = Rcos a.

When squaring both sides

$k^2\sin^2 a + k^2\cos^2 a = 0.1^2 + 0.1^2$.

$k^2(\sin^2 a + \cos^2 a) = 0.02$.

$k^2 = 0.02$.

$k = 0.1414 = \sqrt{2}/10$

$\frac{k\sin a}{k\cos a} = \frac{0.1}{0.1} = 1$

Be recall, sin/cos = tan

$\tan a = 1$

$\tan^{-1}(1) = a$.

$a = 45^\circ$ or $\pi/4$ radian

Steady state.

$= \frac{\sqrt{2}}{10}\sin(\epsilon + \pi/4)$