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18/Eng 04/ 080
Electrical/ Electronics Engineering.
ENG 381

Assignment I

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Solution

(a)

Let's assume/ say,

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

using the auxiliary equation.

$\Rightarrow$ $m^2 + 5m + 6 = 0$

$m^2 + 3m + 2m + 6 = 0$

$m(m+3) + 2(m+3) = 0$

$(m+3)(m+2) = 0$

$m+3 = 0$, $m+2 = 0$

$m_1 = -3$ & $m_2 = -2$

$$x = Ae^{m_1 t} + Be^{m_2 t}$$

$$x = Ae^{-3t} + Be^{-2t} \quad \text{——— C.F}$$

P.I $\quad f(t) = \cos t$

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

Substituting, we have;

$\Rightarrow -C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$\Rightarrow -C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6C\cos t + 6D\sin t = \cos t$

$\Rightarrow -C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$\Rightarrow -C\cos t + 5D\cos t + 6C\cos t - D\sin t - 5C\sin t + 6D\sin t = \cos t$

$= \cos t[-C + 5D + 6C] + \sin t[-D - 5C + 6D] = \cos t$

Let's say;

$= \cos t[-C + 5D + 6C] + \sin t[-D - 5C + 6D] = 1\times\cos t + 0\times\sin t$

$\therefore \Rightarrow -C + 5D + 6C = 1$

$-D - 5C + 6D = 0$

$\Rightarrow \cancel{5D}\ 5C + 5D = 1$

$-5C + 5D = 0$

Solving, we have;

$5C = 5D$

from $\therefore$ $5C + 5D = 1$

$\Rightarrow 5D + 5D = 1 \quad \Rightarrow 10D = 1$

$\therefore D = 1/10$

$\therefore \Rightarrow 5C = 5(1/10)$

$5C = 1/2$

$C = \frac{1/2}{5}$

$C = 1/10$

$\therefore C = 1/10,\ D = 1/10$

$\therefore$ P.I, $\cancel{x}\ \bar{x} \Rightarrow \frac{1}{10}\cos t + \frac{1}{10}\sin t \Rightarrow \frac{1}{10}(\cos t + \sin t)$

G.S = P.I + C.F

$GS \equiv x$

$\therefore GS = Ae^{-2t} + Be^{-3t} + \frac{1}{10}(\cos t + \sin t)$.

recall, when $x = 0.1$, $t = 0$

$\therefore \Rightarrow 0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}(\cos(0) + \sin(0))$

$= 0.1 = A + B + \frac{1}{10}$.

$0.1 = A + B + 0.1$

$A + B = 0.1 - 0.1$

$A + B = 0$ ——— (1).

also, recall $\frac{dx}{dt} = 0$, $t = 0$.

$\Rightarrow x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}(\cos t + \sin t)$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + \frac{1}{10}(-\sin t + \cos t)$

$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} + \frac{1}{10}(-\sin(0) + \cos(0))$

$0 = -2A - 3B + 0.1$

$\Rightarrow 2A + 3B = 0.1$ ——— (2).

Solving eqtn (1) & (2).

$B = -A$ & $A = -B$.

$\therefore 2A + 3B = 0.1$

$= 2(-B) + 3B = 0.1 \Rightarrow -2B + 3B = 0.1$

$B = 0.1 = \frac{1}{10}$

$\therefore \cancel{B = -A}$ $A = -B$

$\therefore A = -0.1 = -\frac{1}{10}$.

Substituting;

$x = -\frac{1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}(\cos t + \sin t)$ — P.S

(5)

b) Matlab:

- Command window
- clc
- clear
- close all
- syms t
- t = 0 : 0.01 : 15
- x = [-(1/10) * exp(-(2*t))) + (1/10 * exp(-(3*t))) + ...
  ... + 1/10 * (cos(t) + sin(t))
- plot (t, x)
- Grid on
- Grid minor
- axis tight
- y label ('vibrations')
- x label ('time')

(c)

Steady state equation

$x_{ss} = \frac{1}{10} \cos t + \frac{1}{10} \sin t$

$= \frac{1}{10} \cos t + 0.1 \sin t = K \sin(t + a)$

$K \sin(t+a) = K \sin t \cos a + K \cos t \sin a$

$\Rightarrow$ $\cos t$ $\qquad\qquad$ $\sin t$

$= 0.1 = K \sin a \quad ; \quad 0.1 = K \cos a$

Let's square both sides;

$K^2 \sin^2 a + K^2 \cos^2 a = 0.01 + 0.01$

$K^2(\sin^2 a + \cos^2 a) = 0.02$

$K^2 = 0.02 \Rightarrow K = \sqrt{0.02}$

$K = 0.1414$

$\therefore \Rightarrow \frac{K \sin a}{K \cos a} = \frac{0.1}{0.1}$

$\Rightarrow \dfrac{\sin a}{\cos a} = 1$

(recall, $\frac{\sin}{\cos} = \tan$)

$\Rightarrow \tan a = 1$

$a = \tan^{-1}(1)$

$a = 45°$

$(\pi = 180)$ $a = \pi/4$

Steady state is ;

$$x_{ss} = \frac{\sqrt{2}}{10} \sin\left(t + \frac{\pi}{4}\right)$$

graph ...