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17/ENG 03/025

Eng 381

Assignment 1

1a)  $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

$$m^2 + 5m + 6 = 0$$

Using completing the square method.

$$m + 5m + \left(\frac{-5}{2}\right)^2 = -6 + \frac{25}{4}$$

$$(m + 5/2)^2 = 1/4$$

$$(m + 5/2) = \pm 1/2$$

$$m = -3 \text{ or } m = -2$$

$$C.F = Ae^{-3t} + Be^{-2t}$$

Particular integral

$$x = C \cos t + D \sin t$$

$$\frac{dy}{dx} = -C \sin t + D \cos t$$

$$\frac{d^2y}{dx^2} = -C \cos t - D \sin t$$

$$C(-\cos t - D \sin t) + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t \quad \dots \text{eqn (1)}$$

$$-C \cos t + 5D \cos t + 6C \cos t + (-D \sin t - 5C \sin t + 6D \sin t) = \cos t$$

$$-C \cos t - 5C \sin t + 5D \sin t = 0 \quad \dots \text{eqn (2)}$$

$$5C + 5D = 1$$

$$5C + 5D = 0$$

$$10D = 1$$

$$D = 1/10$$

$$5C + 5(1/10) = 1$$

$$C = 1/2 \times 1/5$$

$$C = 1/10$$

$$P.I = x = 1/10 \cos t + 1/10 \sin t$$

$$x = Ae^{-3t} + Be^{-2t} + 1/10(\sin t - \cos t)$$

When  $t=0$ ,  $x=0$  &  $dy/dx=0$

$$0.1 = A(1) + B(1) + 1/10(-1)$$

$$0.1 = A + B - 0.1$$

$$A + B = 0.1 + 0.1$$

$$A + B = 0.2 \text{ ----- equation 3}$$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} + 1/10(\cos t + \sin t)$$

$$0 = -3A - 2B + 1/10(1)$$

$$-0.1 = -3A - 2B$$

$$3A + 2B = 0.1 \text{ --- eqn 4}$$

Thus,

$$A + B = 0.2$$

$$A = 0.2 - B \text{ --- eqn 5}$$

Sub equation (5) into eqn 4

$$3(0.2 - B) + 2B = 0.1$$

$$0.6 - 3B + 2B = 0.1$$

$$-B = -0.5$$

$$B = 0.5$$

$$A = -0.3$$

$$x = -0.3e^{-3t} + 0.1e^{-2t} + 1/10(\sin t - \cos t)$$
$$= 0.1e^{-2t} - 0.3e^{-3t} + 1/10(\sin t - \cos t)$$

B Command window

close all;

clear all;

clc

Sym x, t

t = [0:0.1:1.5]

x = 0.1\*exp(-2\*t) - 0.3\*exp(-3\*t) + 0.1\*(sin(t) - cos(t))

plot(t, x)

1c)  $x = K \sin(\omega t + a)$   
Knowing  $x = 0.1$  at  $t = 0$  &  $dx/dt = 0$

$$\frac{dx}{dt} = K \cos(\omega t + a)$$

at

$$0 = K \cos(0 + a)$$

$$K \cos(a) = 0$$

$$0 = K \sin(0 + a) \quad \text{--- equation}$$

$$K \sin(a) = 0.1$$

$$\cos(a) = 0 \Rightarrow a = \cos^{-1}(0)$$

$$a = 90^\circ$$

Sub(a) into equation 1

$$0.1 = K \sin(90)$$

$$K = 0.1 / \sin 90$$

$$K = 0.1$$

$$x = 0.1 (\sin(\omega t) + 90)$$

1c) Command window

close all;

clear all;

clc;

syms x

t = [0:0.1:15]

x = 0.1 \* (sin(t) + 90)

plot(t, x)