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Computer Engineering

18/ENG02/025

30th September 2019

a. Show that the limit of the function given in equation (1.1) as x approaches 0 is a/b

$$F(x) = \frac{\sin ax}{bx} \quad (1.1)$$

Solution

$$f(x) = \frac{\sin ax}{bx}$$

$$= \frac{\sin a(0)}{b(0)} = \frac{0}{0}$$ {the equation is undefined}.

Using L'hopital's rule.

$$\lim_{x \to 0} f(x) = \frac{\sin ax}{bx}$$

$$= \frac{a \cos ax}{b}$$

$$\lim_{x \to 0} f(x) = \frac{a \cos a(0)}{b} = \frac{a}{b}$$

b. The model of a system has been developed to bx as given in equation (1.2)

$$f(x) = 5x - 21 \quad (1.2)$$

Given that $\delta = 0.1$ and $\Delta\delta = 0.01$, demonstrate, in tabular form, the limit of the model as $x \longrightarrow 6$ is equal to 9.

Solution

$\delta = 0.1$, $\Delta\delta = 0.01$, $\varepsilon = 0.05$

6 - 0.1 = 5.9 (left hand side)

6 + 0.1 = 6.1 (Right hand side)

9 - 0.5 = 8.5 (left hand side)

9 + 0.5 = 9.5 (Right hand side)

x	f(x)
5.9	8.5
5.91	8.55
5.92	8.60
5.93	8.65
5.94	8.70
5.95	8.75
5.96	8.80
5.97	8.85
5.98	8.90
5.99	8.95
6.00	9.00

x	f(x)
6.1	9.5
6.09	9.45
6.08	9.40
6.07	9.35
6.06	9.30
6.05	9.25
6.04	9.20
6.03	9.15
6.02	9.10
6.01	9.05
6.00	9.00

c Show whether the function given in equation 1.3 is continous on the interval $[-5, 5]$

$$F(x) = (25 - x^2)^{1/2} \quad (1.3)$$

Solution.

At $x \rightarrow -5$

$$f(x) = (25 - (-5)^2)^{1/2}$$
$$= (25.25)^{1/2}$$
$$= 0$$

$x \rightarrow -4$

$$f(x) = [25 - (-4)^2]^{1/2}$$
$$f(x) = (9)^{1/2}$$
$$f(x) = 3$$

$x \rightarrow -3$

$$f(x) = [25 - (-3)^2]^{1/2}$$
$$f(x) = (16)^{1/2}$$
$$f(x) = 4$$

$x \rightarrow -2$

$$f(x) = [25 - (-2)^2]^{1/2}$$

$f(x) = (21)^{1/2}$

$f(x) = 4.58$

$x \to -1$

$f(x) = [25 - (-1)^2]^{1/2}$

$f(x) = [25 - 1]^{1/2}$

$f(x) = 4.89$

$x \to 0$

$f(x) = [25 - (0)^2]^{1/2}$

$= (25)^{1/2}$

$= 5$

$x \to 1$

$f(x) = [25 - (1)^2]^{2}$

$= (24)^{1/2}$

$= 4.89$

$x \to 2$

$f(x) = [25 - (2)^2]^{1/2}$

$f(x) = (25 - 4)^{1/2} \quad = (21)^{1/2}$

$f(x) = 4.58$

$x \to 3$

$f(x) = [25 - (3)^2]^{1/2}$

$f(x) = (25 - 9)^{1/2} \quad = (16)^{1/2}$

$f(x) = 4$

$x \to 4$

$f(x) = [25 - (4)^2]^{1/2}$

$f(x) = (9)^{1/2}$

$f(x) = 3$

$x \to 5$

$f(x) = [25 - (5)^2]^{1/2}$

$f(x) = (25 - 25)^{1/2}$

$f(x) = 0$

Therefore, the function on the interval $[-5, 5]$ is continous.