KSSANG ROSEMARY
ELECT/ELECT ENGINEERING
17/ENG04/023

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$$

In auxiliary form

$$C.F = m^2 + 5m + 6 = 0$$
$$m^2 + 2m + 3m + 6 = 0$$
$$m(m+2) + 3(m+2)$$
$$m = -2, \quad m = -3$$

$$\therefore x = Ae^{-2t} + Be^{-3t}$$

$$P.I = \cos t$$
$$x = C\cos t + D\sin t$$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

$$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$$
$$-C\cos t + 6C\cos t - D\sin t + 6D\sin t - 5C\sin t + 5D\cos t = \cos t$$
$$5C\cos t + 5D\sin t + 5D\cos t - 5C\sin t = \cos t$$
$$5C + 5D = 1 \quad \text{---(i)}$$
$$5D - 5C = 0 \quad \text{---(ii)}$$

Using simultaneous equation

$$5C + 5D = 1$$
$$-5C + 5D = 0$$
$$10D = 1$$

$$D = 1/10$$

Sub $D = 1/10$ into eqn 1 (i)

$$5C + 5(1/10) = 1$$
$$5C = 1/2$$
$$C = 1/10$$

$$G.S = C.F + P.I$$
$$= Ae^{-2t} + Be^{-3t} + 1/10(\sin t + \cos t)$$

When $t = 0$, $x = 0.1$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 1/10(\sin 0 + \cos 0)$$
$$0.1 = A + B + 0 + 1/10$$

$$A + B = 0 \quad -(iii)$$

When $t = 0$, $dx/dt = 0$

$$dx/dt = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t)$$

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} + 0.1(\cos(0) - \sin(0))$$
$$0 = -2A - 3B + 0.1$$
$$-0.1 = -2A - 3B$$

recall, $A + B = 0$

$$A = -B \quad ---(iv)$$
$$-0.1 = -2(-3) - 3B$$

$-0.1 = 2B - 3B$

$-0.1 = -B$

$B = 0.1$

From eqn(iv)

$A = -0.1$

$P.S = -0.1e^{-2t} + 0.1e^{-3t} + 1/10(\sin t + \cos t)$

2 ~~Clear~~

2 ~~Command window~~

2

1	Commandwindow
2	Clear
3	clc
4	close all
5	x = [1/10 * exp(-2*t) - (1/10 * exp(-3*t) + (1/10 (sin t) + (cost))]
6	t = 0:0.01:15
7	plot(t, x)
8	x label('t')
9	y label('x')
10	grid on
11	grid minor

3 At steady state

~~t ⟶~~

$t_{x \to 0}$ = steady state = $0.1\cos t + 0 + 8\sin t$

$0.1\cos t + 0.1\sin t = K\sin(t + a)$

~~$K\sin(t+a) = K\sin t \cos a$~~

$K\sin(t+a) = K(\sin t \cos a) + K(\cos t \sin a)$

coefficient of cos t = $K\sin a$

Coefficient of sint = K cos a

When squaring both sides

$K^2 \sin^2 a + K^2 \cos^2 a = 0.1^2 + 0.1^2$

$K^2(\sin^2 a + \cos^2 a) = 0.02$

$K^2 = 0.02$

$K = 0.144 = \sqrt{2}/10$

$\frac{K \sin a}{K \cos a} = \frac{0.1}{0.1} = 1$

recall sin/cos = tan

$\tan a = 1$

$\tan^{-1}(1) = a$

$a = 45°$ or $\pi/4$ radian

Steady state

$= \sqrt{2}/10 \sin(t + \pi/4)$