

NAME: BE ORIGINAL ANSW

MATRIC NO: 17ENIG05/039

DEPARTMENT: MECHANICAL ENGINEERING

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos(t)$$

Given that $t=0$, $x=0.1$ $\frac{dx}{dt}=0$

$$CF = m^2 + 5m + 6 = 0$$

$$(m+3)(m+2) = 0$$

$$m = -3, m = -2$$

$$x = Ae^{-3t} + Be^{-2t}$$

Particular solution

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

Subs.

$$= -C \cos t - D \sin t + 5(C \cos t + D \sin t) + 6(C \cos t + D \sin t) = \cos t$$

$$= \cos t (-C + 5D + 6C) + \sin t (C - D - 5D + 6D) = \cos t$$

$$= \cos t (5C + 5D) + \sin t (C - 5C + 5D) = \cos t$$

$$5D + 5C = 1$$

$$5D - 5C = 0$$

$$WD = I \quad ; \quad D = \frac{1}{10}$$

find C

$$5C + \frac{1}{10} = 1$$

$$\frac{1}{10} + 5C = 1$$

$$C = \frac{9}{50}$$

$$G.S. = x = Ae^{-3t} + Be^{-2t} + \frac{9}{50} \cos t + \frac{1}{10} \sin t$$

Subs. $(x=0)$ $t=0$

$$0 = Ae^{-0} + Be^{-0} + \frac{9}{50} \cos 0 + \frac{1}{10} \sin 0$$

$$0 = A + B + 0.18$$

$$A + B = 0 \quad \text{--- eqn 1}$$

$$\frac{dx}{dt} = -3xe^{-2t} - 2ye^{-2t} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

$$0 = -3A - 2B + \frac{1}{10}$$

$$3A + 2B = \frac{1}{10} \quad \text{--- eqn (1)}$$

$$AB = 0 \quad \times 2$$

$$3A + 2B = 0 \quad \times 1$$

$$2A + 2B = 0$$

$$3A + 2B = 0.1$$

$$-A = -0.1$$

$$A = 0.1$$

To find B

$$0.1 + B = 0$$

$$B = -\frac{1}{10} \text{ or } -0.1$$

$$x = \frac{1}{10} e^{-2t} + \frac{1}{10} e^{-2t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$y = \frac{1}{10} e^{-2t} - \frac{1}{10} e^{-2t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$= \frac{1}{10} (e^{-2t} - e^{-2t} + \cos t + \sin t)$$

• MATLAB mfile

Command window

clear

clc

close all

syms t

t = 0:0.01:15

x = 0.1 * [exp(-2*t) - exp(-3*t) + cos(t) + sin(t)]

y = 0.1 * [exp(-2*t) - exp(-3*t) + cos(t) + sin(t)]

plot(t, x)

axis tight

grid on

grid minor