

Mechanical Engineering

a) $\frac{d^2z}{dt^2} + 5\frac{dz}{dt} = 6z - \cos t$

Auxiliary equation: $m^2 + 5m + 6 = 0$

$$m^2 + 2m + 3m + 6 = 0$$

$$m(m+2) + 3(m+2) = 0$$

$$(m+3)(m+2) = 0$$

$$m = -3 \text{ or } -2$$

Complementary function: $x = Ae^{-3t} + Be^{-2t}$

Particular integral: Assume $z = (C \cos t + D \sin t)$

$$\frac{dz}{dt} = (-C \sin t + D \cos t)$$

$$\frac{d^2z}{dt^2} = (-C \cos t - D \sin t)$$

Putting the value of $\frac{d^2z}{dt^2}$ & $\frac{dz}{dt}$ in the equation

$$-C \cos t - D \sin t + 5(C \cos t + D \sin t) = 6(C \cos t + D \sin t) - \cos t$$

Comparing Coefficient

Coefficient of $\cos t$: $C + 5D + 6C = 1$ ————— ①

Coefficient of $\sin t$: $-D + 5C + 6D = 0$ ————— ②

From eqn (1) $5C + 5D = 1$ ————— ③

From eqn (2) $5D - 5C = 0$ ————— ④

From eqn (1) $5D = 1 - 5C$ ————— ⑤

Put eqn (5) into eqn (4)

$$1 - 5C - 5C = 0$$

$$1 - 10C = 0$$

$$C = \frac{1}{10} \text{ or } 0.1 \quad \therefore 5D = 1 - 5C$$

Put the value of C in eqn (5)

$$5D = 0.5$$

$$D = \frac{0.5}{5} = 0.1$$

Particular solution: $z = Ae^{-3t} + Be^{-2t} + 0.1 \sin t + 0.1 \cos t$

When $t=0$; $z = 0.2 \frac{dz}{dt} = 0$

$$0.1 = A + B + 0.1$$

$$A + B = 0$$

$$y' = -3Ae^{-3t} - 2Be^{-2t} + 0.1 \sin t + 0.1 \cos t$$

$$0 = -3A - 2B + 0.1$$

from eqn (b) $B = -A$

$$0 = -3A - 2(-A) + 0.1$$

$$0 = -A + 0.1$$

and $B = -A$

$$B = 0.1$$

$$\text{General solution} = 0.1e^{-3t} - 0.1e^{-2t} + 0.1 \sin t + 0.1 \cos t$$

2) Command window

clear

clc

close all

syms t

$$x = 0.1 + \exp(-3t) - 0.1 * \exp(-2t) + 0.1 * (\cos(t) + \sin(t))$$

$$t = 0; 0.1 : 1.5$$

$$x_{17} = \text{subs}(x)$$

plot(t, x)

xlabel('time')

ju no

grid minor

axis