ENGINEERING MATHS

ASSIGNMENT

1) The diagram model of a body in motion ~~perform~~ performing damped forced vibrations is as in Equation (1)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that when $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

a) using the Auxiliary equation Method, obtain the solution of the model in form of an expression, namely x as a function of t.

b) with the aid of a MATLAB mfile program, plot the relationship between x and t for $0 \leq t \leq 15$ time unit using a step of 0.01 unit, and

c) write the steady-state solution of the model in form of $x = k\sin(t + q)$

Solution

a) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

Auxiliary Equation: $m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$

$(m+3)(m+2) = 0$

$m = -3$ or -2

complementary function $x = Ae^{-3x} + Be^{-2x}$

Particular Integral : Assume $x = C\cos t + D\sin t$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

Putting the value of d^2x/dt^2 & dx/dt in the equation

$-\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

Cost

Company Coefficient

Coefficient of cost :- $6 + 5D + 6C = 1$ —— (I)

Coefficient of $\sin t$:- $-D - 5C + 6D = 0$ —— (II)

From eqn (I) $5C + 5D = 1$ —— (III)

from eqn (II) $5D - 5C = 0$ —— (IV)

from eqn (III) $5D = 1 - 5C$ —— (V)

Put eqn (V) in eqn (IV)

$$1 - 5C - 5C = 0$$

$$1 - 10C = 0$$

$$C = 1/10, 0.1$$

$\therefore 5D = 1 - 5C$

Put the value of C in eqn (5)

$$5D = 0.5$$

$$D = \frac{0.5}{5} = 0.1$$

particular solution : $x = Ae^{-3t} + Be^{-2t} + 0.1\sin t + 0.1\cos t$

when $t = 0;\ x = 0.2 \quad \frac{dx}{dt} = 0$

$0.1 = A + B + 0.1$

$A + B = 0$ ——— (b)

$dy/dx = -3Ae^{-3t} - 2Be^{-2t} + 0.1\cos t + 0.1\cos t$

$0 = -3A - 2B + 0.1$

from eqn (b) $B = -A$

$0 = -3A - 2(-A) + 0.1$

$0 = -A + 0.1$

and $B = -A$

$B = 0.1$

~~General solution = $0.1e^{-3t} - 0.1e^{-2t} + 0.1\sin t +$~~

General solution = $0.1e^{-3t} - 0.1e^{-2t} + 0.1\sin t + 0.1\cos t$

~~2) Command window~~

Command window

2) ~~clear~~ Clear

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x = 0.1* exp(-3*t) - 0.1* exp(-2*t) + 0.2*cos(t) + 0.1*sin(t)

t = ~~0~~ 0; 0.1 :15

x in s Subs Cs

plot (t, xn)

Wenbel ('time')

grid no

Grid minor

axis font

e $0.1\cos t + 0.2\sin t = k\sin(2t+a)$ at steady flow $0.1\cos t + 0.2\sin t = k\sin\cos$ at $k\sin\cos t$

Company coefficient

Coefficient of $\cos t = 0.1 = k\sin a$

Coefficient at $\sin t = 0.1 = k\sin a$

Square $k\sin a$ and $k\cos a$ and equate it to the addition

$$k^2\sin^2 at + k^2\cos^2 a = 0.1 + 0.1$$

$$k^2\sin^2 at\,(\cos^2 a) = 0.2$$

$$k^2 = 0.2 \qquad (\sin^2 at + \cos^2 a = 1)$$

$$k^2 = \frac{2}{100}$$

$$k = \frac{\sqrt{2}}{10}$$

$$k\sin a = \frac{0.1}{0.1}$$

$$\tan a = 1$$

$$a = \tan^{-1}(1)$$

$$a = 45^\circ \text{ or } \frac{\pi}{4}$$

$\therefore$ In steady state;

$$i_{ss} = \frac{\sqrt{2}}{10} \cdot \sin\left(\pi t + \frac{\pi}{4}\right) \text{ A}$$