

a) Auxiliary equation: $k^2 + sk + 6 = 0$

$$k^2 + 2k + 3k + 6 = 0$$

$$k(k+2) + 3(k+2) = 0$$

$$(k+2)(k+3) = 0$$

$$k = -2 \quad \text{or} \quad k = -3$$

Complementary function: $x = Ae^{-3x} + Be^{-2x}$

Particular Integral: assume $x = C \cos t + D \sin t$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

Substituting:

$$-C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t + 5D \cos t + 6C \cos t - D \sin t - 5C \sin t + 6D \sin t = \cos t + 0$$

$$5D \cos t + 5C \cos t = \cos t$$

$$5C + 5D = 1 \quad \dots \dots \textcircled{i}$$

$$-5C \sin t + 5D \sin t = 0$$

$$5D - 5C = 0$$

$$5D = 5C \quad \dots \dots \textcircled{ii}$$

\therefore from equation \textcircled{i} :

$$5C + 5C = 1$$

$$10C = 1$$

$$C = \frac{1}{10} ; 0.1$$

from equation \textcircled{ii} :

$$5D = 5C = 5(0.1)$$

$$5D = 0.5$$

$$D = \frac{0.5}{5}$$

$$D = 0.1$$

Particular Integral: $x = Ae^{-3x} + Be^{-2x} + 0.1 \sin t + 0.1 \cos t$

When $t=0$; $x=0.1$, $dx/dt=0$

c) c

$$x = Ae^{-3t} + Be^{-2t} + 0.1 \sin t + 0.1 \cos t$$

$$0.1 = A + B + 0.1$$

$$A + B = 0 \quad \dots \textcircled{i}$$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} + 0.1 \cos t - 0.1 \sin t$$

$$0 = -3Ae^{-3t} - 2Be^{-2t} - 0.1 \sin t + 0.1 \cos t$$

at $t=0$

$$0 = -3A - 2B + 0.1$$

$$3A + 2B = 0.1$$

From eq. (i):

$$A + B = 0$$

$$\therefore A = -B$$

$$3(-B) + 2B = 0.1$$

$$-3B + 2B = 0.1$$

$$-B = 0.1 \quad ; \quad B = -0.1$$

$$\text{Since } A = -B$$

$$A = 0.1$$

\therefore General solution: $0.1e^{-3t} - 0.1e^{-2t} + 0.1 \sin t + 0.1 \cos t$

b) Command window

clear

clc

close all

syms x t

$$x = 0.1 * \exp(-3*t) - 0.1 * \exp(-2*t) + 0.1 * \cos(t) + 0.1 * \sin(t)$$

$$t = 0:0.01:15$$

$$K_n = \text{linspace}(x)$$

Plot (t, K_n)

K_label('time')

Grid on

Grid minor

c) $0.1 \cos t + 0.1 \sin t = k \sin(\omega t + a)$ at steady state
 $0.1 \cos t + 0.1 \sin t = k \sin \omega t \cos a + k \cos \omega t \sin a$

Comparing coefficient

coefficient of $\cos t$: $0.1 = k \cos a$

coefficient of $\sin t$: $0.1 = k \sin a$

Square $k \sin a$ and $k \cos a$ and equate it to the addition

$$k^2 \sin^2 a + k^2 \cos^2 a = 0.1 + 0.1$$

$$k^2 (\sin^2 a + \cos^2 a) = 0.2 \quad [\sin^2 a + \cos^2 a = 1]$$

$$k^2 = 0.2 = \frac{2}{10}$$

$$k = \frac{\sqrt{2}}{10}$$

$$\frac{k \sin a}{k \cos a} = \frac{0.1}{0.1}$$

$$\tan a = 1$$

$$a = \tan^{-1}(1)$$

$$a = 45^\circ \text{ or } \frac{\pi}{4}$$

k steady state

$$k_{\text{set}} k_{sc} = \frac{\sqrt{2}}{10} \sin\left(\frac{\pi}{4} + t\right)$$