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 Computer Engineering
 Assignment 1

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t \quad \text{--- (1)}$$

auxiliary equation

$$m^2 + 5m + 6 = 0$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$(m+3)(m+2) = 0$$

$$m = -3 \text{ or } -2$$

$$CF = Ae^{-3t} + Be^{-2t}$$

$$PI = x = C \cos t + D \sin t \quad \text{--- (2)}$$

$$\frac{dx}{dt} = -C \sin t + D \cos t \quad \text{--- (3)}$$

$$\frac{d^2x}{dt^2} = -C \cos t + (-D \sin t) \quad \text{--- (4)}$$

Put (2), (3) & (4) in eqn (1)

$$-C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

Taking coefficient of $\cos t$

$$-C + 5D + 6C = 1 \quad \text{--- (5)}$$

$$5D + 5C = 1 \quad \text{--- (6)}$$

Taking coefficient of $\sin t$

$$-D - 5C + 6D = 0 \quad \text{--- (7)}$$

$$5D - 5C = 0 \quad \text{--- (8)}$$

from equation (8)

$$5D = 5C \quad \text{--- (9)}$$

Put (8) in (6)

$$(1-5c) - 5c = 0$$

$$1 - 5c - 5c = 0$$

$$1 - 10c = 0$$

$$1 = 10c$$

$$c = \frac{1}{10} = 0.1$$

Put (6) in (5)

$$5D = 1 - 5\left(\frac{1}{10}\right)$$

$$5D = \frac{1}{2} \Rightarrow D = \frac{1}{10}$$

$$D = \frac{1}{10}$$

$$0 = 0.1 \cos t + 0.1 \sin t$$

PI $\Rightarrow x = 0.1 \cos t + 0.1 \sin t$

Particular solution

$$x = A e^{-3t} + B e^{-2t} + 0.1 \cos t + 0.1 \sin t$$

when $t = 0, x = 0.1$

$$0.1 = A + B + 0.1 \cos 0 = A + B + 0.1$$

$$0 = A + B - 0.1$$

$$A = -B$$

when $t = 0, \frac{dx}{dt} = 0$

$$\frac{dx}{dt} = -3A e^{-3t} - 2B e^{-2t} - 0.1 \sin t + 0.1 \cos t$$

$$0 = -3A - 2B - 0 + 0.1$$

$$0 = -3A - 2B + 0.1$$

$$0 = -3(-B) - 2B + 0.1$$

$$0 = B + 0.1$$

Put (9) in (8)

$$A = -(-0.1)$$

$$A = 0.1$$

General solution

$$x = 0.1 e^{-3t} + (-0.1) e^{-2t} + 0.1 \cos t + 0.1 \sin t$$

$$x = 0.1 e^{-3t} - 0.1 e^{-2t} + 0.1 \cos t + 0.1 \sin t$$

Matlab

Command window

clc

clear all

close all

syms x,t

t = 0:0.01:15

x = 0.1 * exp(-3*t) - 0.1 * exp(-2*t) + 0.1 * cos t + 0.1 * sin t

xn = subs(x)

plot(t, xn)

xlabel('Time')

grid on

grid minor

axis tight

② $0.1 \cos t + 0.1 \sin t = K \sin(t + \alpha)$ at steady state

$$0.1 \cos t + 0.1 \sin t = K \sin \cos \alpha + K \cos t \sin \alpha$$

Comparing Coefficient

Coefficient of $\cos t$

$$0.1 = K \sin \alpha$$

Coefficient of $\sin t$

$$0.1 = K \cos \alpha$$

Assume

$$K^2 \sin^2 \alpha + K^2 \cos^2 \alpha = 0.1^2 + 0.1^2$$

$$K^2 = 0.02$$

$$K = \sqrt{0.02}$$

$$= \sqrt{2/100}$$

$$\frac{K \sin \alpha}{K \cos \alpha} = \frac{0.1}{0.1} = 1$$

$$\tan \alpha = 1 \quad \alpha = \tan^{-1}(1)$$

$$\alpha = \pi/4 = 45^\circ$$

$$x \text{ at steady state ; } \frac{\sqrt{2}}{10} \sin \left[\frac{\pi}{4} + t \right]$$

