

$$1) \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$m^2 + 5m + 6 = 0$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$m+2=0, m+3=0$$

$$m = -2 \text{ or } m = -3$$

$$\therefore \text{C.f} = Ae^{-2t} + Be^{-3t}$$

$$\text{P.I} = \cos t$$

$$x = (\cos t + D \sin t)$$

$$\frac{dx}{dt} = -(\sin t + D \cos t)$$

$$\frac{d^2x}{dt^2} = -(\cos t + D \sin t)$$

$$= -(\cos t + D \sin t) + 5(-\sin t + D \cos t) + 6(\cos t + D \sin t)$$

$$= \cos t$$

$$= -(\cos t - D \sin t - 5 \cos t + 5D \sin t + 6 \cos t + 6D \sin t)$$

$$= \cos t \cos t (-1 + 5 + 6) + (-D + 5D + 6D) \sin t$$

$$= \cos t \cos t + 10D \sin t = \cos t$$

$$10D \sin t - 10D \sin t = 0$$

$$10D = 1$$

$$D = 1/10$$

$$D = 1/10$$

$$D = 1/10$$

$$5C + 5D = 1$$

$$5C + 5 = 1$$

$$C = 1/10$$

$$\therefore \text{P.I} = 1/10 (\cos t + \sin t)$$

$$\therefore x = \text{C.F} + \text{P.I}$$

$$x = Ae^{-2t} + Be^{-3t} + 1/10 (\cos t + \sin t)$$

$$\text{when } t=0, x=0.1 \text{ and } \frac{dx}{dt} = 0$$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 1/10 (\cos 0 + \sin 0)$$

$$0.1 = A + B - 0.1$$

$$A + B = 0.1 + 0.1$$

$$A + B = 0.2 \dots \dots \text{equation (1)}$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + 1/10 (\cos t + \sin t)$$

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} + 20/10(1)$$

$$= -2A - 3B + 0.1$$

$$= 3A + 2B = 0.1 \dots \dots (2)$$

$$A + B = 0.2$$

$$A = 0.2 - B \dots \dots (3)$$

$$\text{Sub equation (3) into equation (2)}$$

$$3(0.2 - B) + 2B = 0.1$$

$$0.6 - 3B + 2B = 0.1$$

$$-B = -0.5$$

$$B = 0.5$$

$$A = -0.3$$

$$\therefore x = -0.3e^{-2t} + 0.5e^{-3t} + 1/10 (\cos t + \sin t)$$

b) Command Window

clear

clc

close all

Sym x; t.

t = [0:0.1:1.5]

x = 0.1 * sin(t + 90);

plot(t, x)

c) $x = K \sin(t + \alpha)$

$$x = 0.1 \quad t = 0 \quad dx/dt = 0$$

$$\frac{dx}{dt} = K \cos(t + \alpha)$$

$$0 = K \cos(0 + \alpha)$$

$$0 = K \cos \alpha \quad \dots (1)$$

$$0.1 = K \sin(\alpha) \quad \dots (2)$$

$$K \sin \alpha = 0.1$$

$$\cos \alpha = 0 \quad \alpha = \cos^{-1}(0)$$

$$\alpha = 90^\circ$$

Sub into equation 2

$$0.1 = K \sin 90^\circ$$

$$K = 0.1 / \sin 90^\circ$$

$$K = 0.1$$

$$x = 0.1 \sin(t + 90^\circ)$$