

Darrabadi Ibanga Aains

17EN021029

Computer Engineering

ENUB81

Assignment 1

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$m^2 + 5m + 6 = 0 \quad \text{--- Auxiliary Equation}$$

$$m^2 + 2m + 3m + 6 = 0$$

$$m(m+2) + 3(m+2)$$

$$(m+2)(m+3)$$

$$m = -2 \text{ or } m = -3$$

$$x = Ae^{-2t} + Be^{-3t}$$

For particular Integral,

$$x = C\cos t + D\sin t$$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

$$-C\cos t - D\sin t - 5C\cos t - 5D\sin t + 6C\cos t + 6D\sin t = \cos t$$

$$5C + 5D = 1$$

$$5D - 5C = 0$$

$$5D = 5C$$

$$D = C$$

$$5D + 5D = 1$$

$$10D = 1$$

$$D = \frac{1}{10}$$

$$C = \frac{1}{10}$$

General solution.

$$x = Ae^{-2t} + Be^{-3t} + 0.1\cos t + 0.1\sin t$$

$$\text{at } x = 0.1 \quad \frac{dx}{dt} = 0 \quad \text{at } t = 0$$

$$A + B = 0$$

$$A = -B$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - 0.1\sin t + 0.1\cos t$$

$$0 = -2A - 3B + 0.1$$

$$2A + 3B = 0.1$$

$$-2B + 3B = 0.1$$

$$B = 0.1$$

$$A = -0.1$$

Particular solution

$$x = -0.1e^{-2t} + 0.1e^{-3t} + 0.1\cos t + 0.1\sin t$$

2. Command window

clear

clc

close all

syms t

$$x = (1/10 * \exp(-2*t)) - (1/10 * \exp(-3*t)) + (1/10 * (\sin t) + (\cos t))$$

$$t = 0:0.01:15$$

$$xt = \text{subs}(x, t)$$

$$xtn = \text{double}(xt)$$

Plot(t, xtn)

xlabel('t')

ylabel('x')

grid on

grid minor

grid right

$$8. x_{ss} = x(t \rightarrow \infty) \Rightarrow 0.1\cos t + 0.1\sin t = k \sin(t + a)$$

$$x_{ss} = k \sin t \cos a + k \cos t \sin a$$

$$0.1 = k \sin a$$

$$0.1 = k \cos a$$

$$k^2 \sin^2 a + k^2 \cos^2 a = 0.01 + 0.01$$

$$k^2 (\sin^2 a + \cos^2 a) = 0.02$$

$$k^2 = 0.02$$

$$k = \sqrt{2}/\omega$$

$$k \sin a = 0.1 \Rightarrow \sqrt{2} \sin a = 0.1$$

$$k \cos a = 0.1 \Rightarrow \sqrt{2} \cos a = 0.1$$

$$\tan a = 1$$

$$a = \pi/4$$

$$x_{ss} = \frac{\sqrt{2}}{10} \sin(t + \pi/4)$$

