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CHEMICAL ENGINEERING
17/ENG01/011
ENG 381

ASSIGNMENT

1) Solution

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

In auxillary form,

$$m^2 + 5m + 6 = 0$$

$$(m+2)(m+3) + 6$$

$$m+2 = 0 \quad m+3 = 0$$

$$m_1 = -2 \quad m_2 = -3$$

$$x = Ae^{-2x} + Be^{-3x}$$

P.I = cost

$$x = C\cos t + D\sin t$$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$

Sub into the equation

$$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$$

$$\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$$

$$-C\cos t + 6\cos t - D\sin t + 6D\sin t - 5C\sin t + 5D\cos t = \cos t$$

$$5C\cos t + 5D\cos t + 5D\sin t - 5C\sin t = \cos t$$

$$5C\cos t + 5D\cos t = \cos t$$

$$5C + 5D = 1 \quad \text{---- (1)}$$

$$5D\sin t - 5C\cos t = 0$$

$$5D - 5C = 0 \quad \text{---- (2)}$$

Using simultaneous equation

$$5C + 5D = 1 \quad \text{---- (1)}$$

$$-5C + 5D = 0 \quad \text{---- (2)}$$

$$10D = 1$$

$D = 1/10$

$5c + 5(1/10) = 1$

$5c + 1/2 = 1$

$5c = 1 - 1/2$

$5c = 1/2$

$c = 1/10$

General solution

$x = Ae^{-2x} + Be^{-3x} + 1/10\,[\sin t + \cos t]$

Where $t = 0 \quad x = 0.1$

$0.1 = Ae^{-2(0.1)} + Be^{-3(0.1)} + 1/10\,[\sin(0) + \cos(0)]$

$0.1 = A + B + 0 + 1/10$

$A + B \text{ ---- } (i)$

When $t = 0 \quad dx/dt = 0$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t)$

$0 = -2A - 3B + 0.1$

$-0.1 = -2A - 3B \text{ --- } (ii)$

Recall $A + B = 0 \text{ ---- } (iii)$

$A = -B \text{ --- } (iv)$

$-0.1 = -2(-B) - 3B$

$-0.1 = 2B - 3B$

$-0.1 = -B$

$B = 0.1$

Recall that $A = -B$

$A = -0.1$

$\therefore x = -0.1e^{-2t} + 0.1e^{-3t} + 1/10\,[\sin t + \cos t]$

2. Command Window

```
Clear
clc
Close all
Syms t
x=(1/10*exp(-2*t) -(1/10*exp(-3*t) + (1/10*(sin(t) + cos(t)))
t = 0:0.1:15
xt = subs(x,t)
```

```
xtn = double(xt)
plot(t, xtn)
  x label('t')
  y label('x')
  grid on
  grid minor
  grid tight
```

At steady state

$x = x_{ss} = 0.1\cos t + 0.1\sin t$

$t \to \infty$ steady state

$0.1\cos t + 0.1\sin t = k\sin(t+\alpha)$

$k\sin(t+\alpha) = k\sin t\cos\alpha + k\cos t\sin\alpha$

co-efficient of $\cos t = k\sin\alpha$

co-efficient of $\sin t = k\cos\alpha$

Square both sides

$k^2\sin^2\alpha + k^2\cos^2\alpha = 0.1^2 + 0.1^2$

$k^2(\sin^2\alpha + \cos^2\alpha) = 0.02$

$k^2 = 0.02$

$k = \sqrt{0.02}$

$k = 0.1414 = \sqrt{2}/10$

$\frac{k\sin\alpha}{k\cos\alpha} = \frac{0.1}{0.1} = 1$

Recall that $\frac{\sin}{\cos} = \tan$

$\tan\alpha = 1$

$\tan^{-1}(1) = \alpha$

$\alpha = 45^\circ$ or $\alpha = \pi/4$ radian

Steady state:

$= \frac{\sqrt{2}}{10}\sin(t + \pi/4)$