KUNDE SHARON SEPINEN
17/ENG03/032R
CIVIL ENGINEERING
ENG 381

1) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

in auxiliary form

$m^2 + 5m + 6 = 0$

$(m+2)(m+3) = 0$

$m+2 = 0$ or $m+3 = 0$

$x = Ae^{-2t} + Be^{-3t}$

$PI = \cos t$ $\quad x = C\cos t + D\sin t$

$dx/dt = -C\sin t + D\cos t$

$d^2x/dt^2 = -C\cos t + D\sin t$

Sub into the equation

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6[C\cos t + D\sin t] = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t - 6C\cos t + 6D\sin t = \cos t$

$-C\cos t + 6C\cos t - D\sin t + 6D\sin t - 5C\sin t + 5D\cos t = \cos t$

$5C\cos t + 5D\sin t - 5C\sin t + 5D\cos t = \cos t$

$5C\cos t + 5D\cos t + 5D\sin t - 5C\sin t = \cos t$

$5C\cos t + 5D\cos t = \cos t$

$5C + 5D = 1 \quad ---$ ①

$5D\sin t - 5C\sin t = 0$

$5D - 5C = 0$

using simultaneous equation

$5C + 5D = 1 \quad ---$ ①

$-5C + 5D = 0 \quad ---$ ②

$10D = 1$

$D = 1/10$

$5C + 5\left(\frac{1}{10}\right)$

$5C + 1/2 = 1$

$5C = 1 - 1/2$

$5C = 1/2$

$C = 1/10$

General solution

$Ae^{-2t} + Be^{-3t} + \frac{1}{10}[\sin t + \cos t]$

when $t = 0$, $x = 0.1$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}[\sin(0) + \cos(0)]$

$0.1 = A + B + 0 + \frac{1}{10}$

$A + B = 0$ --- ①

When $t = 0$ $dx/dt = 0$

$dx/dt = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t)$

$0 = -2A - 3B + 0.1$

$-0.1 = -2A - 3B$ --- ②

Recall $A + B = 0$ --- (iii)

$A = -B$ --- (iv)

$-0.1 = -2(-B) - 3B$

$-0.1 = 2B - 3B$

$-0.1 = -2(-B) = 3B$

$-0.1 = 2B - 3B$

$-0.1 = -B$

$B = -B$

$B = 0.1$

Recall that $A = -B$

$A = -0.1$

$\therefore x = -0.1e^{-2t} + 0.1e^{-3t} + \frac{1}{10}[\sin t + \cos t]$

2) command window

```
clear
clc
close all
syms t
x = (1/10 * exp(-2*t)) - (1/10 * exp(-3*t)) + (sin(t) + cos(t))
t = 0:0.01:15
xt = subs(x,t)
plot(t, xt+n)
xlabel('t')
ylabel('x')
grid on
grid minor
grid right
```

At steady state

$x = x_{ss} = 0.1\cos t + 0.1\sin t$

$t \to \infty$ steady state

$0.1\cos t + 0.1\sin t = K\sin(t+a)$

$K\sin(t+a) = K\sin t\cos a + K\cos t\sin a$

coefficient of $\cos t = K\sin a$

coefficient of $\sin t = K\cos a$

square both side

$K^2\sin^2 a + K^2\cos^2 a = 0.1^2 + 0.1^2$

$K^2(\sin^2 a + \cos^2 a) = 0.02$

$K^2 = 0.02$

$K = \sqrt{0.02}$

$K = 0.1414 = \sqrt{2}/10$

$\frac{K\sin a}{K\cos a} = \frac{0.1}{0.1} = 1$

Recall that $\sin/\cos = \tan$

$\tan a = 1$

$\tan^{-1}(1) = a$

$a = 45^\circ$ or $\pi/4$ radian

steady state

$= \frac{\sqrt{2}}{10}\sin(t + \pi/4)$