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ENG 331

Q.

The dynamic model of a body in motion performing damped forces Vibration is a 2nd order equation

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$$

Given that $t=0$, $x=0$ and $dx/dt = 0$

a) Using the auxiliary equation method, Obtain the Solution of the model information of an expression having x as a function of t

b) With the aid of matlab m-file program, plot the relationship between x and t for $0 \le t \le 15$ time int using a step size of 0.01 and end

c) Write the Steady State Solution of the model in form of $x = K\sin(t-2)$

Solution

a) $\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$

In auxiliary form

$m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$(m^2+2m)(3m+6)$

$m(m+2)\,3(m+2)$

$m = -2, \; m = -3$

$x = Ae^{-2t} + Be^{-3t}$

$PI = \cos t$

$x = C\cos t + D\sin t$

$dx/dt = -C\sin t + D\cos t$

$d^2x/dt^2 = -C\cos t - D\sin t$

Substituting back into equation

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 6D\cos t + 6C\cos t + 6D\sin t = \cos t$

$-C\cos t + 6C\cos t - D\sin t + 6D\sin t - 5C\sin t + 5D\cos t = \cos t$

$5C\cos t + 5D\sin t - 5C\sin t + 5D\cos t = \cos t$

$5c\cos t + 5D\cos t + 5D\sin t - 5c\sin t = \cos t$

$5c + 5D = 1$ ... (i)

$5D\sin t - 5c = 0$

$5D - 5c = 0$ ... (ii)

Using Simultaneous Equation

$5c + 5D = 1$

$-5c + 5D = 0$

$10D = 1$

$D = 1/10$

$5c + 5(1/10) = 1$

$5c + 1/2 = 1$

$5c = 1 - 1/2$

$5c = 1/2$

$c = 1/10$

$G.S = Ae^{-2t} + Be^{-3t} + 1/10[\sin t + \cos t]$

when $t = 0$, $x = 0.1$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + 1/10[\sin(0) + \cos(0)]$

$0.1 = A + B + 0 + 1/10$

$A + B = 0$ ... (i)

when $t = 0$, $dx/dt = 0$

$dx/dt = -2Ae^{-2t} - 3Be^{-3t} + 0.1(\cos t - \sin t)$

$0 = -2A - 3B + 0.1$

$-0.1 = -2A - 3B$

Remember $A + B = 0$ ... (iii)

$A = -B$ (iii)

$-0.1 = -2(-B) - 3B$

$-0.1 = 2B - 3B$

$-0.1 = -B$

$B = 0.1$

Knowing that $A = -B$

$A = -0.1$

$x = -0.1e^{-3t} + 0.1e^{-3t} + \frac{1}{10}[\sin t + \cos t]$ OR

$x = \frac{-1}{10}e^{-3t} + \frac{1}{10}e^{-3t} + \frac{1}{10}[\sin t + \cos t]$

2) Command Window

```
Clear
Clc
Close all
Syms t
x = (1/10 * exp(-2*t)) - (1/10 * exp(-3*t)) + (1/10 (Sin(t) + Cos(t)))
t = 0:0.01:15
xt = subs (x,t)
xtn = double (xt)
Plot (t, x, tn)
x label ('t')
y label ('x')
grid on
grid minor
grid right
```

3. At Steady State

$x_{t\to 0} = x_{\text{steady state}} = 0.1\cos t + 0.1\sin t$

$0.1\cos t + 0.1\sin t = K\sin(t+n)$

$K\sin(t+a) = K\sin + \cos a + K\cos t \sin a$

NB Coefficient of $\cos t = K\sin a$

" " $\sin t = K\cos a$

When Squaring both Sides

$K^2\sin^2 a + K^2\cos^2 a = 0.1^2 + 0.1^2$

$K^2(\sin^2 a + \cos^2 a) = 0.02$

$K^2 = 0.02$

$K = \sqrt{0.02}$.

$K = 0.144 = \sqrt{2}/10$

$\frac{K \sin\theta}{K \cos\theta} = \frac{0.1}{0.1} = 1$

Remember that $\sin/\cos = \tan$

$\tan a = 1$

$\tan^{-1}(1) = a$

$a = 45^\circ$ or $\pi/4$ radian

Steady.

$= \sqrt{2}/10 \sin(t + \pi/2)$