Name: Inyang Godwin OKON
Matric NO: 17/ENG01/015
Dept: Chemical ENGINEERING
ENG 381

1) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$ at $t=0$, $x=0.1$; $\frac{dx}{dt} = 0$

Solution

$m^2 + 5m + 6 = 0$ → Assuming $f(x) = 0$

$m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$

$(m+2)(m+3) = 0$

$m_1 = -2$ & $m_2 = -3$

$C.F = x = Ae^{m_1 x} + Be^{m_2 x}$

$x = Ae^{-2t} + Be^{-3t}$

$P.I = f(x) = \cos t$

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$= -C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$= -C\cos t + 5D\cos t + 6C\cos t - D\sin t - 5C\sin t + 6D\sin t = \cos t$

$= (-C + 5D + 6C)\cos t + (-5C - D + 6D)\sin t = \cos t$

$\therefore (5C + 5D)\cos t + (5C + 5D)\sin t = \cos t + D\sin t$

$\rightarrow 5C + 5D = 1$ —— (1)

$+ \quad -5C + 5D = 0$ —— (2)

Solving simultaneously

$10D = 1$

$\rightarrow D = \frac{1}{10}$

Sub $D = \frac{1}{10}$ into equ 1.

$5C + 5(1/10) = 1$

$5C + 1/2 = 1$

$5C = 1/2$

$10C = 1$

$C = \frac{1}{10}$

$P.I = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$G.S = \frac{1}{10}(\cos t + \sin t) + Ae^{-2x} + Be^{-3x}$.

$x = \frac{1}{10}(\cos t + \sin t) + Ae^{-2t} + Be^{-3t}$

$0.1 = 1/10(\cos t + \sin t) + Ae^{0} + Be^{0}$

$0.1 = 1/10(1+0) + A + B$

$0.1 = \frac{1}{10} + A + B$

$\rightarrow A + B = 0$ — (1)

$x = \frac{1}{10}(\cos t + \sin t) + Ae^{2t} + Be^{-2t}$

$\frac{dx}{dt} = 1/10(-\sin t + \cos t) - 2Ae^{-2t} - 3Be^{-3t}$

$0 = \frac{1}{10} - 2A - 3B$

$-2A - 3B = -1/10$ — (2)

$A + B = 0$ — (1)

$-2A - 3B = -1/10$ — (2)

$2A + 2B = 0$

$-2A - 3B = -1/10$

$-B = -1/10$

$B = 1/10$

Put $B = 1/10$ into eqn 1

$A + 1/10 = 0$

$A = -1/10$

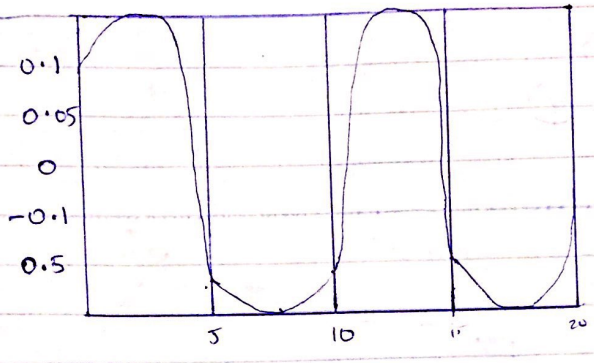
$x = \frac{1}{10}\cos t + \frac{1}{10}\sin t - \frac{1}{10}e^{-2t} + \frac{1}{10}e^{-3t}$

$x = \frac{1}{10}(\cos t + \sin t - e^{-2t} + e^{-3t}$

1b) MATLAB

Command window

```
clear
clc
clear all
syms t
t = 0:0.01:15
x = 0.1*(exp(-3*t) - exp(-2*t) + (cos(t) + sin(t)))
xn = subs(x)
plot(t, xn)
axis tight
grid on
grid minor
```



$x = K\sin(t+10)$

Knowing that $x=0.1$, $t=0$ & $\frac{dx}{dt}=0$,

$0 = K\cos(0+a)$

$K\cos(a) = 0$

$0.1 = K\sin(0+a)$

$K\sin(a) = 0.1$ ... (1)

$\cos a = 0$

$a = \cos^{-1} = 0$

$= 40^{\circ}$ //

Sub a into (1)

$0.1 = K\sin(90)$

$\therefore K = \frac{0.1}{\sin 90} = 0.1$

$x = 0.1(\sin(t+90))$

Command window

```
close all
Clear
clc
Syms t, x
   t=(0:0.01:15)
    x=0.1*(sin(t+90))
    plot(t,x)
```