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17/ENG01022
COMPUTER ENGINEERING
ENG381

1. $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

A.E: $k^2 + 5k + 0 = 0$

$k^2 + 2k + 3k + 6 = 0$

$k(k+2) + 3(k+2) = 0$

$(k+3)(k+2) = 0$

$k_1 = -3 \quad k_2 = -2$

$x_{c.f} = Ae^{-3t} + Be^{-2t}$

P.I:

$f(x) = \cos t$

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$(-\cos t - D\sin t) - 5C\sin t + 5D\cos t + 6C\cos t$

$\cos t: \quad -C + 5D + 6C = 1$

$5C + 5D = 1$

$\sin t: \quad -D - 5C + 6D = 0$

$-5C + 5D = 0 \quad \text{---} \ (2)$

Eqn (1)+(2) $\quad 10D = 1$

$D = \frac{1}{10}$

Sub D into (2)

$5C + 5(\frac{1}{10}) = 1$

$5C = 1 - \frac{1}{2}$

$C = \frac{1}{2} \times \frac{1}{5}$

$= 1/10$

$x_{P.I} = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$x_{G.S} = Ae^{-3t} + Be^{-2t} + \frac{1}{10}[\cos t + \sin t]$

When $t=0$, $x=0.1$ and $\frac{dx}{dt}=0$

$0.1 = A(1) + B(1) + \frac{1}{10}(1)$

$0.1 = A + B + 0.1$

$A + B = 0.1 - 0.1$

$A + B = 0$ ---- ①

$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - \frac{1}{10}\sin t + \frac{1}{2}\cos t$

$0 = -3A - 2B + \frac{1}{10}$ --- ②

$A + B = 0$ --- $\times 2$

$3A + 2B = 0.1$ --- $\times 1$

$2A + 2B = 0$

$3A + 2B = 0.1$

$-A = -0.1$

$\therefore A = 0.1$

To find B

$0.1 + B = 0$

$B = -0.1$

$x_{c.s} = \frac{1}{10}e^{-3t} - \frac{1}{10}e^{-2t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$= \frac{1}{10}[e^{-3t} - e^{-2t} + \cos t + \sin t]$

MATLAB

Command window

clear

cls

Clear all

Syms t

$x = 0.1 * [\exp(-3*t)] - \exp(-2*t) + \cos(t) + \sin(t)$

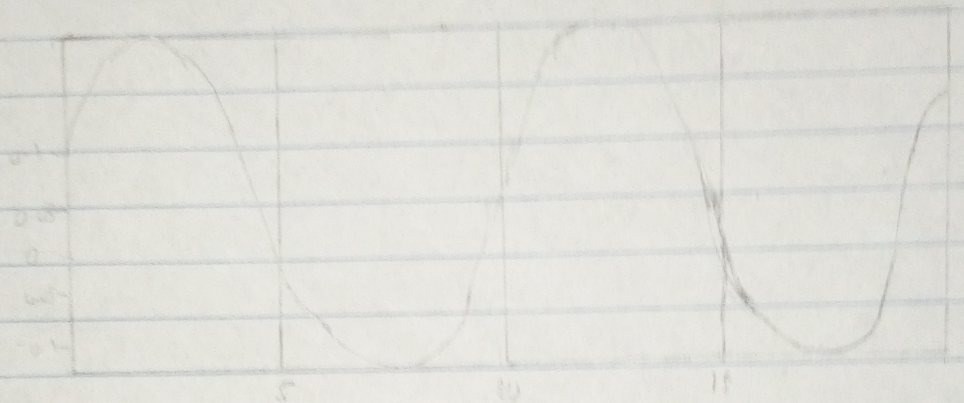
xn = subs (x)

plot (t, xn)

axis tight

grid on

grid minor



c. $x = k\sin(t+\gamma)$

knowing $x = 0.1$ at $t = 0$ & $\frac{dx}{dt} = 0$

$\frac{dx}{dt} = k\cos(t+\alpha)$

$0 = k\cos(0+\alpha)$

$k\cos(\alpha) = 0$

$0.1 = k\sin(0+\alpha)$

$k\sin(\alpha) = 0.1$

$\cos(\alpha) = 0$

$\alpha = \cos^{-1} = 0$

$= 90$

Sub α into (1)

$k\sin(90) = 0.1$

$k = \frac{0.1}{\sin 90} = k = 0.1$

$x = 0.1[\sin(t+90)]$