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MECHANICAL SYSTEM

The damped model of a body in motion performing damped forces vibration is a sin equation

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$$

Given that $t=0, x=0.1$ and $dx/dt = 0$

a) Using the analytical method. Obtain the solution of the model information of an expression having x as a function of t.

b) With the aid of matlab mfile program. plot the relationship between x and t for $0 \le t \le 1.5$ time diff in t using a step size of 0.01 start end

c) Write the steady state equation of the model inform of $x = k\sin(t-\alpha)$

Solution

① $\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$

for CF where $dx/dt = a$, $x = 1$, $f(t) = 0$

$$a^2 + 5a + 6 = 0$$

$[3, 2]$

$$a^2 + 3a + 2a + 6 = 0$$

$$a(a+3) + 2(a+3) = 0$$

$$(a+2)(a+3)$$

$a = -2$, $a = -3$, real & different

$f(t) = Ae^{-2t} + Be^{-3t}$ //

for P.I

$= \cos t$

where $x = C\cos t + D\sin t$

$dx/dt = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$\therefore$ Let $x = \cos t$, $\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

where $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

$(-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$\cos t(-C + 5D + 6C) + \sin t(-D - 5C + 6D) = \cos t$

$\therefore \sin t(5D - 5C) = 0$

$5D - 5C = 0$

$5D = 5C$

$D = C$

$\cos t(-C + 5D + 6C) = \cos t$

$5C + 5D = 1$

where $C = D$

$5D + 5D = 1$

$10D = 1$

$\therefore D = 1/10 \quad C = 1/10$

$x = 1/10(\cos t + \sin t)$

$\therefore$ General Solution

$y = C.F + P.I$

$x = Ae^{-2t} + Be^{-3t} + 1/10(\cos t + \sin t)$ //

when $t = 0$, $x = 0.1$

$\frac{1}{10} = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}(\cos(0) + \sin(0))$

$\frac{1}{10} = A + B + \frac{1}{10}$

$\frac{1}{10} - \frac{1}{10} = A + B \quad \therefore A + B = 0 \longrightarrow (i)$

when $dx/dt = 0$, $x = 0.1$

$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$

where $dx/dt = 0$, $x = 0.1$, $t = 0$

$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{1}{10}\sin(0) + \frac{1}{10}\cos(0)$

$-\left(0 = -2A - 3B + \frac{1}{10}\right) \quad = \quad 0 = 2A + 3B - \frac{1}{10}$

$10(2A + 3B) = \left(\frac{1}{10}\right)10$

$20A + 30B = 1 \longrightarrow (ii)$

$A + B = 0 \Rightarrow (iv)$

$A = -B \longrightarrow (iv)$

Put iii in ii

$20(-B) + 30B = 1$

$-20B + 30B = 1$

$10B = 1, \quad B = \frac{1}{10} \quad A = -\frac{1}{10}$

$\therefore x = \frac{1}{10}\left(e^{-3t} - e^{-2t} + \cos t + \sin t\right)$

b Command window

```
clc
close all
Sysm t
x = 1/10*exp(-3*t) - 1/10*exp(-2*t) + (cos(t)+(sin(t))
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```
t = 0 : 0.01 : 15
xt = subs (x, t)
x+n = double (xt)
plot (t, x+n)
x label ('t')
y label ('x')
grid on
grid minor
grid right
```

Q At steady state

$x \to x_c$, $t \to 0$ = steady state $= 0.1\cos t + 0.1\sin t$

$0.1\cos t + 0.1\sin t = K\sin(t+a)$

$K\sin(t+a) = K\sin t\cos a + K\cos t\sin a$