Eremasara Ernest
17/ENG04/022
Electrical / Electronic
17/ENG04/022
ENG 381

1. $\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$

A.E:
$$k^2 + 5k + 6 = 0$$
$$k^2 + 2k + 3k + 6 = 0$$
$$k(k+2) + 3(k+2) = 0$$
$$(k+3)(k+2) = 0$$
$$k_1 = -3 \text{ or } k_2 = -2$$
$$x_{c.f} = Ae^{-3x} + Be^{-2x}$$

P.I:
$$F(x) = \cos t$$
$$x = C\cos t + D\sin t$$
$$\frac{dx}{dt} = -C\sin t + D\cos t$$
$$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$$
$$[-C\cos t - D\sin t] - 5C\sin t + 5D\cos t + 6C\cos t - 6D\sin t = \cos t$$

$\cos t$: $-C + 5D + 6C = 1$
$$5C + 5D = 1 \quad \text{---} \quad (1)$$

$\sin t$: $-D - 5C + 6D = 0$
$$-5C + 5D = 0 \quad \text{---} \quad (2)$$

Eqn (1) + (2): $10D = 1$
$$D = \frac{1}{10}$$

Subst D into (1)
$$5C + 5\left[\frac{1}{10}\right] = 1$$
$$5C = 1 - \frac{1}{2}$$
$$C = \frac{1}{2} \times \frac{1}{5}$$
$$= \frac{1}{10}$$

Command window

```
Close all
Clear
CLC
Syms t, x
    t = [0:0.01:15]
    x = 0.1 [sin (t + 90)]
plot (t, x)
```

$x_{p.I} = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$x_{c.s} = Ae^{-3t} + Be^{-2t} + \frac{1}{10}[\cos t + \sin t]$

when $t=0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

$0.1 = A(1) + B(1) + \frac{1}{10}(1)$

$0.1 = A + B + 0.1$

$A + B = 0.1 - 0.1$

$A + B = 0 \quad \text{---} (1)$

$dx/dt = -3Ae^{-3t} - 2Be^{-2t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$

$0 = -3A - 2B + \frac{1}{10} \quad \text{---} (2)$

$A + B = 0 \quad \text{---} \times 2$

$3A + 2B = 0.1 \quad \text{---} \times 1$

$2A + 2B = 0$

$3A + 2B = 0.1$

$-A = -0.1$

$A = 0.1$

To find B

$0.1 + B = 0$

$B = -\frac{1}{10}$

$x_{c.s} = \frac{1}{10}e^{-3t} - \frac{1}{10}e^{-2t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$= \frac{1}{10}[e^{-3t} - e^{-2t} + \cos t + \sin t]$

(iii) MATLAB

command window

Clear

CLC

```
close all
syms t
t = 0:0.01:15
x = 0.1*[exp(-3*t)] - exp(-2*t) + cos(t) + sin(t)

xn = subs(x)
plot (t, xn)
axis tight
grid on
grid minor
```

c. $x = h\sin(t+a)$

Knowing $x = 0.1$ at $t = 0$ & $dx/dt = 0$

$$\frac{dx}{dt} = h\cos(t+a)$$

$$0 = h\cos(0+a)$$

$$h\cos(a) = 0$$

$$0.1 = h\sin(0+a)$$

$$h\sin(a) = 0.1 \quad \text{- - - (1)}$$

$$\cos(a) = 0$$

$$a = \cos^{-1} = 0$$

$$= 90^\circ$$

Sub a into (1)

$$h\sin(90) = 0.1$$

$$h = 0.1/\sin 90 \Rightarrow h = 0.1$$

$$x = 0.1[\sin(t+90)]$$