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Assignment 1,

1) the dynamic mode of a body in motion performing damped force vibration is an equation (1).

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos 8t.$$

Given that when $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

a) using the auxillary equation, obtain the solution of the model in form of an expression having x as a function of t.

b) with aid of MATLab mfile program plot the relationship between x and t for $0 \leq t \leq 18$ time units using a step size of 0.01 unit and.

c) write the steady state solution of the model in from of $x = k \sin(t + a)$.

Solution

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t.$$

Auxillary equation $m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$

$(m+3)(m+2) = 0$

$m = -3$ or -2

Complementary function $x = Ae^{-3t} + Be^{-2t}$

P.I = Assume $x = C\cos t + D\sin t$

$$dx/dt = \sin t + D\cos t.$$

$$d^2x/dt^2 = -C\cos t - D\sin t$$

Putting the value of d^2x/dt^2 & dx/dt in the equation

$$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$$

$$C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t.$$

$$-C\cos t + 5D\cos t + 6C\cos t - D\sin t - 5C\sin t + 6D\sin t = \cos t.$$

Coefficient of $\cos t$: $-C + 5D + 6C = 1$ — (1)

Coefficient of $\sin t$: $-D - 5C + 6D = 0$ — (2)

from equation (1) = $5C + 5D = 1$ ——— (3)

from equation (2) = $5D - 5C = 0$ ——— (4)

from equation (3) = $5D = 1 - 5C$

Put equation (3) in (4)

$$= 1 - 5C - 5C = 0$$

$$1 - 10C = 0$$

$$10C = 1$$

$$C = 1/10 \quad \text{——— (5).}$$

Put equ 5 in 4.

$$5D - 1/2 = D.$$

$$D = 0.1.$$

General solution : $x = Ae^{-3t} + Be^{-3t} + 0.1\cos t + 0.1\sin t.$

Particular Solution when $t=0$ $x=0.1$ $\frac{dx}{dt}=0$

$0.1 = A + B + 0.1$

$A + B = 0$

$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - 0.1\sin t + 0.1\cos t.$

$0 = -3A - 2B + 0.1 \quad ---(2)$

from equation (1).

$B = -A \quad ---(3)$

Put equation 3 in 2

$0 = -3A - 2(-A) + 0.1$

$0 = -3A + 2A + 0.1$

$0 = -A + 0.1$

$A = 0.1$

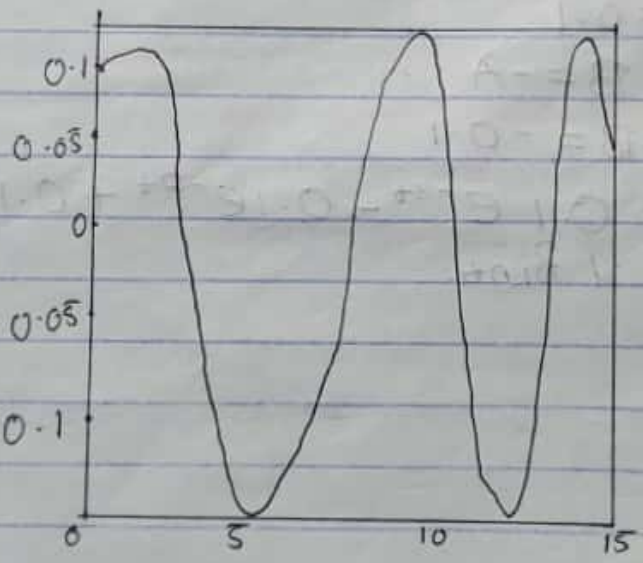
and $B = -A$

so $B = -0.1$

General Solution $= 0.1e^{-3t} - 0.1e^{-2t} + 0.1\cos t + 0.1\sin t.$

b) Command window.

```
Clear
Clc
Close all
Syms x t
x = 0.1 exp(-3*t) - 0.1 exp(-2*t) + 0.1*cos(t) +
0.1 * sin(t)
t = 0; 0.01 = 15
xn = subs(x)
Plot(t, xn).
x label('time').
grid On
grid major.
axis tight.
```



c) ~~0.1 cos t + 0.1 sin t = k sin(t+a) at steady flow.~~
~~0.1 cos t + 0.1 sin t = k sin t cos a + k sin a cos t.~~
Comparing Coefficient.
Comparing

c) $0.1\cos t + 0.1\sin t = K\sin(t+a)$ at steady flow

$0.1\cos t + 0.1\sin t = K\sin t\cos a + K\sin a\cos t$

Comparing coefficient

Comparing of coefficient

Coefficient of $\cos t = 0.1 = K\sin a$

coefficient of $\sin t = 0.1 = K\cos a$

Square $K\sin a$ and $K\cos a$ and equate it to the addition

$K^2\sin^2 a + K^2\cos^2 a = 0.1 + 0.1$

$K^2(\sin^2 a + \cos^2 a) = 0.2$

$K^2 = 0.2$

$K^2 = \frac{2}{10}$

$K = \sqrt{\frac{2}{10}}$

$\frac{K\sin a}{K\cos a} = \frac{0.1}{0.1}$

From $\tan a = 1$

$a = \tan^{-1}(1)$

$a = 45^\circ$ or $\frac{\pi}{4}$

$\therefore K$ steady state; $K_{ss} = \sqrt{\frac{2}{10}}\sin\left(\frac{\pi}{4} + t\right)$