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MATRIC NO; LF/ENGE 03/008

DEPT; Civil Engineering

COURSE; ENGE 301

1) The dynamic model of a body in motion performing damped force vibrations is an eqn¹

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos(2t)$$

Given that when $t=0, x=0, dx/dt=0$

2) Using the auxiliary equation method, obtain the solution of the model in form of an expression having x as a function of t ,

3) With the aid of a MATLAB m-file program, plot the relationship between x as a function and t for $0 \leq t \leq 15$ time unit using a step size of 0.01 unit and

4) Write the steady state solution of the model in form of

$$x = 16 \sin(2t + \theta)$$

Solution

$$C-F = m^2 + 5m + 6 = 0$$

$$(m+3)(m+2) = 0$$

$$m = -3, m = -2$$

$$x = Ae^{-3t} + Be^{-2t}$$

$$f(t) = \cos t$$

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

Sub

$$= -C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t)$$

$$D(\sin t) = \cos t$$

$$= \cos t (-C + 5D) + \sin t (-D - 5C + 6D) = \cos t$$
$$= \cos t (5D + 5C) + \sin t (-5C + 5D) = \cos t$$

$$5D + 5C = 1$$

$$5D - 5C = 0$$

$$10D = 1$$

$$D = 1/10$$

to find C

$$5\left(\frac{1}{10}\right) + 5C = 1$$

$$\frac{1}{2} + 5C = 1$$

$$C = \frac{1}{10}$$

$$G.S = x = Ae^{-3t} + Be^{-2t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

Subs $(x=0.1)$ at $t=0$

$$0.1 = Ae^{-0} + Be^{-0} + \frac{1}{10} \cos 0 + \frac{1}{10} \sin 0$$

$$0.1 = A + B + 0.1$$

$$A + B = 0 \quad \text{--- eqn (1)}$$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

$$0 = -3A - 2B + \frac{1}{10}$$

$$3A + 2B = \frac{1}{10} \quad \text{--- eqn (11)}$$

$$A + B = 0 \quad \text{--- (2)}$$

$$3A + 2B = 0.1 \quad \text{--- (2)}$$

$$2A + 2B = 0$$

$$3A + 2B = 0.1$$

$$-A = -0.1$$

$$A = 0.1$$

to find B

$$0.1 + B = 0$$

$$B = -\frac{1}{10} \text{ or } -0.1$$

$$G.S = \frac{-1}{10} e^{-3t} + \frac{1}{10} e^{-2t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$G.S = \frac{1}{10} e^{-2t} - \frac{1}{10} e^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$\frac{1}{10} (e^{-2t} - e^{-3t} + \cos t + \sin t)$$

11) MATLAB mfile

Command window

clear

clc

close all

Syms t

$$t = 0:0.01:15$$

$$x = 0.1 * [\exp(-2*t) - \exp(-3*t) + \cos(t) + \sin(t)]$$

$$x_n = \text{subs}(x)$$

plot(t, x_n)

axis tight

grid on

grid minor