OGAH JOSHUA IMHOAGENE
17/ENG02/063
COMPUTER ENGINEERING

1. $\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$

$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = 0$

$m^2 + 5m + 6 = 0$

$(m+2)(m+3) = 0$

$m = -2$ or $m = -3$

$x = Ae^{-2t} + Be^{-3t}$

$\cos t \rightarrow x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$-5D + 5C = 1$ -- eqn(1)

$5D - 5C = 0$ -- eqn(2)

$10C = 1$ $\quad C = 0.1$

put $C = 0.1$ into eqn (1)

$5D + 5(0.1) = 1$

$5D = 1 - 0.5$

$D = \frac{0.5}{5} = 0.1$

$x = 0.1\cos t + 0.1\sin t$

General Solution, $x = Ae^{-2t} + Be^{-3t} + 0.1\cos t + 0.1\sin t$

when $t = 0$, $x = 0.1$

$0.1 = A + B + 0.1$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - 0.1\sin t + 0.1\cos t$$

when $t=0$, $\frac{dx}{dt} = 0$

$$0 = -2A - 3B + 0.1$$

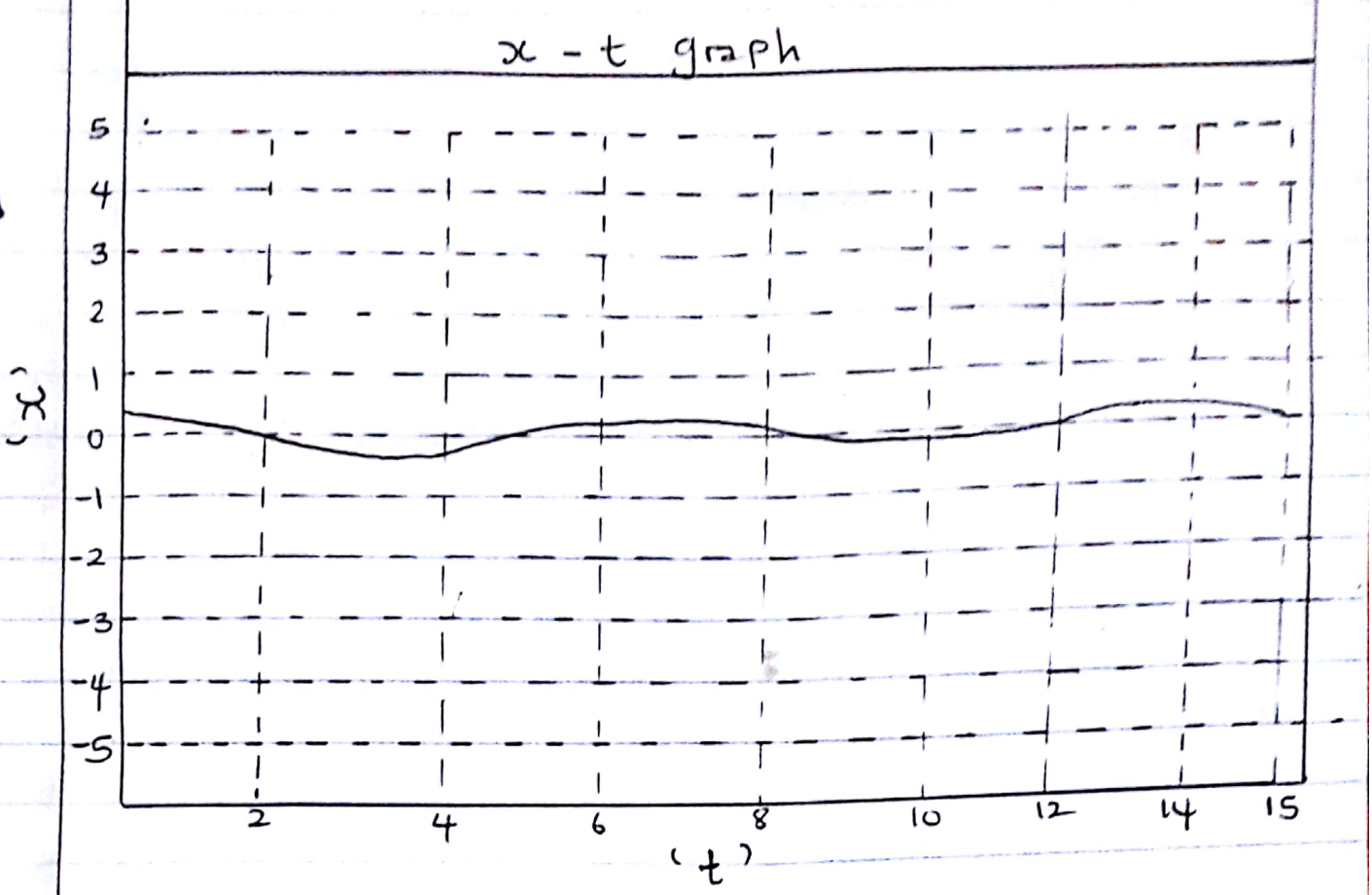
$$A + B = 0 \quad ---(1)$$
$$-2A - 3B = +0.1 \quad ---(2)$$

$$A = -B$$
$$-2(-B) - 3B = -0.1$$
$$-B = -0.1$$
$$B = 0.1$$
$$A + 0.1 = 0$$
$$A = -0.1$$

$$x = -0.1e^{-2t} + 0.1e^{-3t} + 0.1\cos t + 0.1\sin t$$

2.

```
Command Window
Clear
clc
close all
Syms x
Syms t
t = 0:0.01:15
x = 0.1*exp(-2*t) + 0.1*exp(-3*t) + 0.1*(cos(t)
    + 0.1*sin(t)
plot (t,x)
xlabel ('t'), ylabel ('x'), title ('x-t graph')
grid on
axis equal
```



3. $x_{\substack{\text{steady}\\\text{state}}} = x_{t\to\infty} = 0.1\cos t + 0.1\sin t$

$k\sin(t+a) = k\sin t\cos a + k\cos t\sin a$

$0.1\cos t + 0.1\sin t = k\sin t\cos a + k\cos t\sin a$

$0.1 = k\sin a$

$0.1 = k\cos a$

Square both sides

$k^2\sin^2 a + k^2\cos^2 a = \frac{1}{100} + \frac{1}{100}$

$k^2(\sin^2 a + \cos^2 a) = 1/50$

$k^2 = \frac{\sqrt{2}}{10}$

$\frac{k\sin a}{k\cos a} = \frac{0.1}{0.1} = 1$

$\tan a = 1 \quad \therefore \quad a = \pi/4$

$x_{ss} = \frac{\sqrt{2}}{10}\left(t + \frac{\pi}{4}\right)$