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17/ENG06/002
MECHANICAL ENGR
ENG 381

1. $\frac{d^2x}{dt^3} + 5\frac{dx}{dt} + 6x = \cos t$

when $t = 0$, $x = 0.1$ and $dx/dt = 0$

Soln

a) Using auxiliary eqn method,

$m^2 + 5m + 6 = 0$

by factorizing

$(m+3)(m+2) = 0$

$\therefore m_1 = -3$ and $m_2 = -2$

$\therefore x = Ae^{-3t} + Be^{-2t}$

for P.I

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

put it into the original eqn.

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

equating coefficients

$-C + 5D + 6C = 1$ - - - - (i)

$-D - 5C + 6D = 0$ - - - - - (ii)

$5D + 5C = 1$ - - - - - (i)

$5D * -5C = 0$ - - - - - (ii)

subtract eqn (ii) from (i)

$10C = 1$

$C = 1/10$

from eqn (i)

$$5D + 5\left(\frac{1}{10}\right) = 1$$

$$5D + \frac{1}{2} = 1$$

$$5D = \frac{1}{2}$$

$$D = 1/10$$

$\therefore x = P.I = 1/10\,[\cos t + \sin t]$

$\therefore x = C.F + P.I = Ae^{-3t} + Be^{-2t} + 1/10\,[\cos t + \sin t]$

when $t = 0$, $x = 0.1$

$$0.1 = Ae^0 + Be^0 + \frac{1}{10}[\cos 0 + \sin 0]$$

$$0.1 = A + B + 0.1$$

$$0 = A + B$$

$$A = -B \quad - - - - \text{(iii)}$$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} + \frac{1}{10}[-\sin t + \cos t]$$

when $t = 0$, $dx/dt = 0$

$$0 = -3A - 2B + 1/10$$

$$0 = -3A - 2B + 0.1$$

$$-0.1 = -3A - 2B \quad - - - - \text{(iv)}$$

but from eqn (iii), $A = -B$

$$-0.1 = -3(-B) - 2B$$

$$-0.1 = +3B - 2B$$

$$-0.1 = B$$

$\therefore A = -(-0.1) = 0.1$

$\therefore x = 0.1e^{-3t} - 0.1e^{-2t} + 0.1\,[\cos t + \sin t]$

2) Command window

```
clear
clc
close all
syms x, t
x = [0.1 * exp(-3*t)] - [0.1 * exp(-2*t)] + [0.1 * (cost) + sint]
t = 0 : 0.01 : 15
kn = subs (x)
plot (t, kn)
k label ['time']
grid on
grid minor
axis tight
```

3) At - steady state,

$0.1\cos t + 0.1\sin t = k\sin(t + a)$

$0.1\cos t + 0.1\sin t = k\sin t\cos a + k\sin a\cos t$

Comparing coefficients,

for cost, $0.1 = k\sin a$ - - - - (i)

for sint, $0.1 = k\cos a$ - - - - (ii)

Square ksina and kcosa and equate it to 0.1 + 0.1

$k^2\sin^2 a + k^2\cos^2 a = 0.1 * 0.1$

$k^2\sin^2 a + k^2\cos^2 a = 0.2$

$k^2(\sin^2 a + \cos^2 a) = 0.2$

$k^2 = 0.2$ $\quad [\sin^2 a + \cos^2 a = 1]$

$k = \frac{\sqrt{2}}{10}$

for a,

$\frac{k\sin a}{k\cos a} = \frac{0.1}{0.1}$

$\tan a = 1$

$a = \tan^{-1}[1]$

$a = 45^\circ$ or $\pi/4$

∴ k steady state,

$k_{ss} = \frac{\sqrt{2}}{10}\sin(\pi/4 + t)$